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RADIOMETER PRESSURE AND COEFFICIENT OF ACCOMMODATION

BY

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Abstract.

In the present paper radiometer forces have been investigated under such conditions that all the quantities on which the radiometer force may be conceived to depend have been measured, more especially the differences of temperature. The importance of the coefficient of accommodation for the radiometer pressure is shown, and from measurements at low pressures it is found that, in the case under investigation, the coefficient of accommodation for the internal energy (rotational energy) of a diatomic gas may be put equal to the coefficient of accommodation for the translation energy.

Introduction.

On radiometer forces there exists a very extensive literature partly of a theoretical and partly of an experimental kind, but the experiments performed have frequently been carried out in such a way that very complicated phenomena were involved, while, in addition, some very essential quantity was not measured because it was impossible with the experimental arrangements employed.

In a previous work ("Ein absolutes Manometer"[1]) I have examined a particularly simple case under such conditions that the theory could be fully tested by experiments allowing of the measurement of both the temperature and pressure of the gas, and the forces with which it acted on a plate. The experimental arrangement was such that the coefficient of accommodation did not affect the results.

[1] Ann. d. Phys. Bd. 32, 1910, S. 809.

In the experiments to be described in the following, an investigation of the influence of the coefficient of accommodation for the radiometer pressure has been attempted, and here again an endeavour has been made to choose such a simple experimental arrangement that the theory could be tested by the experiments, all the quantities entering into the theoretical formulas being measured.

As is generally known, the specific heat of hydrogen decreases with decreasing temperature. The explanation given of this phenomenon is that as the temperature gradually decreases, the hydrogen molecules or some of them, tend less and less to change their internal energy, particularly their rotational energy, by collision. Now, when it is known that upon impact of hydrogen molecules with a bright metal surface, the exchange of energy will lack $^2/_3$ of its value in being complete, it might be anticipated that, at low temperatures, the exchange of energy would be relatively greater for the translational than for the rotational energy. The experiments show that this anticipation is not justified, at any rate not at such high temperatures as those employed in the experiments. This result seems quite natural now, that it is known that two modifications of hydrogen exist.

Summary of Experimental Results.

The experiments were performed with a thin, narrow plate or band of platinum, bright on one side and blackened on the other. It was placed in an extensive quantity of gas and heated electrically to a higher temperature T_1 than the gas. Its temperature T_1 was measured, and the temperature T_0 and pressure p of the gas far from the band were likewise measured. In addition the amount of heat q' given off each second from each square cm. of the blackened side, and the amount of heat q'' similarly given off from the bright side, were measured. In these amounts the radiation is not included. Although the band has practically the same temperature on both sides, the pressures p' and p'' of the gas on the two sides will nevertheless be different. Their difference $p'-p''$ is termed the radiometer pressure, and this quantity is measured. It is shown that the radiometer pressure will in this case be the same as it would be if the surface had been exactly the same on the two sides of the band, but there had been a difference in temperature of a known magnitude determined by the measurements.

On a previous occasion I have shown, by measuring the molecular conduction of heat in gases, that an incomplete exchange of energy takes place when gas molecules impinge on a solid wall. In order to explain this phenomenon in more detail I introduced the term "coefficient of accommodation". That this quantity, or at any rate a quite analogous quantity, must also play a part in radiometer forces is quite simply a consequence of the kinetic theory.

The coefficient of accommodation a, which is of importance in the conduction of heat, I have defined as the ratio between two temperature differences

$$a = \frac{T_2 - T_0}{T_1 - T_0},$$

where T_0 denotes the temperature of the gas molecules impinging on the surface of a solid body having the temperature T_1. T_2 denotes the temperature of the gas molecules after striking and again leaving the surface of the solid body (Fig. 1).

The investigations to be described in the following were carried out with helium as a representative of a monatomic gas and with hydrogen as a representative of a diatomic gas.

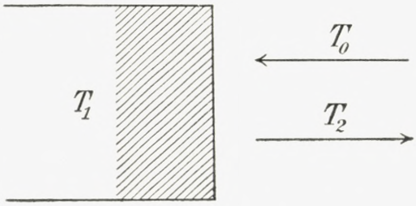

Fig. 1.

Investigation at low Pressures. At such low pressures that the breadth of the band is negligible compared with the mean free path of the surrounding gas we may, according to the kinetic theory, put

$$q = \frac{1}{2} p \bar{c}_0 \frac{T_1 - T_0}{T_0} a \qquad \text{for a monatomic gas and}$$

$$q = \frac{1}{2} p \bar{c}_0 \frac{T_1 - T_0}{T_0} \left(1 + \frac{3}{4} f\right) a \quad \text{for a diatomic gas,}$$

where q is the amount of heat given off per sec. per square cm. in ergs, p the pressure of the gas in bars, $\bar{c}_0$ is the mean velocity of its molecules far from the heated plate, i. e. in those places where the absolute temperature of the gas is T_0. T_1 is the absolute temperature of the plate, and a is the coefficient of accommodation of the gas and surface under consideration. f is determined by the specific heats c_p and c_v for the gas, since $f = \dfrac{\frac{5}{3} - \frac{c_p}{c_v}}{\frac{c_p}{c_v} - 1}$, which is zero for a monatomic gas. For a diatomic gas the formula for q only holds good on the supposition that the coefficient of accommodation a for the translational energy of the gas may be put equal to the coefficient of accommodation for the internal energy of the gas molecules.

Substituting in the formulas $\bar{c_0} = 14550\sqrt{\frac{T_0}{M}}$, where M is the molecular weight of the gas, and putting for helium $f = 0$, and $M = 4.000$, and for hydrogen $f = 0.6325$ and $M = 2.016$, the formulas for q will be

$$\text{for hydrogen } q = \frac{7555}{\sqrt{T_0}}(T_1 - T_0)\,pa,$$

$$\text{for helium } q = \frac{3637.5}{\sqrt{T_0}}(T_1 - T_0)\,pa.$$

Employing for the measurements a platinum band or strip blackened on one side and bright on the other T_1, T_0, p and $q' + q''$ can be measured, from which is found by means of the above formulas, $a' + a''$ the sum of the coefficients of accommodation for the two sides of the band. By corresponding measurements of a band bright on both sides, q'' is obtained, and thence a'' is determined. $a' + a''$ and a'' being known, we find from these $a' - a''$, which is the difference found by heat conduction experiments between the coefficients of accommodation for the blackened and bright sides of the plate.

For the corresponding difference between the coefficients of accommodation a'_t and a''_t which only concern the translational energy of the gas molecules and not their internal energy, the kinetic theory gives for small values of $T_1 - T_0$

$$p' - p'' = p\frac{T_1 - T_0}{4\,T_0}(a'_t - a''_t),$$

from which we find $a'_t - a''_t$, the radiometer pressure $p' - p''$ and p, T_1 and T_0 being measured directly.

The results of the measurements were

for hydrogen:

heat conduction	$a' = 0.735$	$a'' = 0.315$	$a' - a'' = 0.420$
radiometer force			$a'_t - a''_t = 0.415$

for helium:

heat conduction	$a' = 0.909$	$a'' = 0.411$	$a' - a'' = 0.498$
radiometer force			$a'_t - a''_t = 0.512$

The measurements having thus practically given $a' - a'' = a'_t - a''_t$, not only for helium but also for hydrogen, it may be inferred from the measurements for this latter gas that the translational energy and the internal energy have coefficients of accommodation which do not appreciably differ.

The term "coefficient of accommodation" may be extended to apply not only to such large quantities of gas molecules that a temperature may be ascribed to them, but also to single molecules impinging with constant velocity upon the surface of a solid body. If this be done, the measurements provide a means of calculating $\frac{n'}{n''}$, where n' denotes the average number of impacts of a molecule against a rough surface from the first time it impinges on a part of it till it again leaves the surface, and where n'' stands for the same for a bright (smooth) surface. This consideration leads to

$$\frac{n'}{n''} = \frac{\log(1 - a')}{\log(1 - a'')}.$$

Substituting the values for a' and a'' given above, we find

for hydrogen $\frac{n'}{n''} = 3.5$ and for helium $\frac{n'}{n''} = 4.5$.

It might be anticipated that $\frac{n'}{n''}$ must be independent of the gas employed for the determination. The values 4.5 and 3.5 found for helium and hydrogen differ considerably, so that the theory put forward is not particularly well supported by the measurements. It should, however, be borne in mind that, especially when a approximates to 1, the percentage error of $1-a$ entering into the formula will be very considerable.

The agreement found for hydrogen between the value $a'-a''=0.420$, found by heat conduction experiments, and the value $a'_t-a''_t=0.415$, found by radiometer measurement, would seem to warrant the assumption that the internal energy of the hydrogen molecules is distributed according to Maxwell's law, but entirely independent of the distribution of the translational energy.

Investigations at high pressures. It is repeatedly stated in the literature (thus by W. H. Westphal) that when the radiometer pressure $p'-p''$ is plotted against the logarithm of the pressure p of the gas in a rectangular system of co-ordinates, a curve will be found which is very nearly symmetrical with respect to the ordinate through the maximum point of the curve. The formula $p'-p''=\dfrac{1}{\dfrac{1}{ap}+bp}$ has been set up to denote this dependency of pressure. My measurements, too, show that the symmetry in question applies with such good approximation that I have not found it necessary to set up an unsymmetrical formula which agrees better with the experimental results than a symmetrical one.

In the following symmetrical formula which agrees fairly well with the experimental results for helium and hydrogen, $p'-p''$ denotes the radiometer pressure, p the pressure of the gas itself, and T_1 the absolute temperature of the plate, T_0 that of the surroundings. $a_t'-a_t''$ is the difference between the coefficients of accommodation of the two sides of the plate.

$$p'-p'' = \frac{\frac{1}{2}p\left(\sqrt{\frac{T_1}{T_0}}-1\right)(a_t'-a_t'')}{1+b_1 p+c_1 p^2},$$

where $a_t'-a_t''$, b_1 and c_1 are almost independent on the temperature T_1. For small values of T_1-T_0 was found for helium $a_t'-a_t'' = 0.4898$ and for hydrogen 0.363, for helium $b_1 = 0.00545$, $\sqrt{c_1} = 0.005234$, for hydrogen $b_1 = 0.00819$ and $\sqrt{c_1} = 0.00744$. To extend the application of the equation to other gases and to a breadth B of the band other than $B = 0.2484$ cm. as used in the experiment may be tried in the following way.

At high pressures p the radiometer force is known to be an edge effect and we must consequently have c_1 proportional to B. If B be the only length characterising the dimensions of the apparatus, then the unit for this length must be some length characterising the gas f. i. λ_1, the mean free path at the pressure 1 Bar. In putting $c_1 = c\frac{B}{\lambda_1}$, the equation for small values of T_1-T_0 becomes

$$\frac{p'-p''}{T_1-T_0} = \frac{1}{4\,T_0}(a_t'-a_t'')\frac{1}{\sqrt{c\frac{B}{\lambda_1}}}\cdot\frac{1}{\frac{1}{p\sqrt{\frac{cB}{\lambda_1}}}+p\sqrt{\frac{cB}{\lambda_1}}+\frac{b_1}{\sqrt{c\frac{B}{\lambda_1}}}},$$

where for helium $\sqrt{c} = 0.0105$ and for hydrogen $\sqrt{c} = 0.0149$.

According to this equation the radiometer pressure attains its maximum when $p\sqrt{\frac{cB}{\lambda_1}} = 1$, and the maximum value is determined by

$$\left(\frac{p'-p''}{T_1-T_0}\right)_{\max} = \frac{1}{4\,T_0}(a'_t - a''_t)\frac{1}{2\sqrt{\frac{cB}{\lambda_1}} + b_1}.$$

By my experiments the quantities b_1 and $\sqrt{c_1}$ were found to be but little different. If we put $b_1 = \sqrt{c_1} = \sqrt{\frac{cB}{\lambda_1}}$ the equations become

$$\frac{p'-p''}{T_1-T_0} = \frac{1}{4\,T_0}(a'_t - a''_t)\frac{1}{\sqrt{\frac{cB}{\lambda_1}}}\cdot\frac{1}{1 + p\sqrt{\frac{cB}{\lambda_1}} + \frac{1}{p\sqrt{\frac{cB}{\lambda_1}}}}$$

$$\text{and}\quad \left(\frac{p'-p''}{T_1-T_0}\right)_{\max} = \frac{1}{12\,T_0}(a'_t - a''_t)\frac{1}{\sqrt{\frac{cB}{\lambda_1}}}.$$

Hence, to obtain the greatest radiometer force possible with a plate of a given area, it will be advantageous to divide the area into strips which are as narrow as possible.

Calculation of the Loss of Heat at Low Pressures.

Let it be assumed that a platinum band many times longer than it is broad is stretched out in the axis of a hollow cylinder, the walls of which are absolutely rough, and the radius of which is large compared to the breadth of the band. The amount of heat measured in erg given off by conduction through the gas from each square cm. of one surface of the band is designated q', and that given off from each square cm. of the other surface is termed q''.

q is used as a common designation for both these quantities.

Further, the following terms are introduced

p = the pressure in bars of the gas at the surrounding cylinder wall,

N = the number of molecules in each ccm. in the places where the pressure is p,

dN = the number of the N molecules having velocities between c and $c+dc$. This number dN is assumed to be determined by Maxwell's law of the distribution of velocities.

According to the kinetic theory of gases the number of molecules d^2n impinging on each square cm. of the surface of the body each second, and having angles of incidence situated in the solid angle $d\omega$ which forms the angle x with the surface normal, will be

$$d^2n = \frac{1}{4\pi} dN \cdot c \cdot \cos x \cdot d\omega.$$

Each of these molecules gives off the amount of energy $\frac{1}{2} mc^2$, when upon impact with the body, it loses its velocity c, the mass of each single molecule being designated m. Hence the d^2n molecules give the body a quantity of heat $\frac{1}{4\pi} dNc \cos x \, d\omega \cdot \frac{1}{2} mc^2$. Since $d\omega = 2\pi \sin x \, dx$, integration with respect to x gives the quantity of heat $\frac{1}{8} dNmc \cdot c^2$. Integration with respect to N according to Maxwell's law of distribution will give $\frac{1}{8} N m \overline{c_0^3}$ where $\overline{c_0^3}$ denotes the mean value of c^3. According to Maxwell's law of distribution we have $\overline{c_0^3} = \frac{1}{2} \pi (\overline{c_0})^3 = \frac{4}{3} \overline{c_0}\, \overline{c_0^2}$, where $\overline{c_0}$ denotes the mean value of c_0, and $\overline{c_0^2}$ denotes the mean value of c_0^2. Introducing this we get that the quantity of heat given off to each square cm. of the body each second by all the gas molecules impinging on it is $q_{t,1} = \frac{1}{6} Nm\overline{c_0}\,\overline{c_0^2}$ the molecules losing their translational energy by the impact. According to the kinetic theory $p = \frac{1}{3} Nm\overline{c_0^2}$ hence

$$q_{t,1} = \frac{1}{2}\, p\overline{c_0}\,.$$

According to the kinetic theory the number of impacts n, that is to say, the number of impacts experienced by each square cm. each second, is determined by $n = \frac{1}{4} N\overline{c_0}$. Each molecule impinging on the body will thus in mean give off a quantity of heat $\frac{q_{t,1}}{n} = \frac{2}{3} m\overline{c_0^2}$. If the body has the same temperature as the surroundings, each molecule must carry away such an average quantity of energy from the body that the whole transfer of heat will be zero, hence the previously found amount of energy $\frac{q_{t,1}}{n}$.

From this it may be inferred that a quite similar expression only with other values for $\overline{c^2}$ must apply when the body has not the same temperature as the surroundings, if only it is assumed that the velocities of the molecules leaving the body are distributed according to the same law as that of the incident molecules, that is, according to Maxwell's law. If the velocities of the departing molecules are termed c_2, each of them will, consequently, carry away from the body an amount of energy whose mean value will be $\frac{2}{3} m\overline{c_2^2}$. Hence at each fully accomplished molecule impact the body will lose in all the amount of energy $\frac{2}{3} m\left(\overline{c_2^2} - \overline{c_0^2}\right)$ and since the number of impacts is $n = \frac{1}{4} N\overline{c_0}$, the whole amount of heat q_t which the body gives off to the surrounding gas from each square cm. and during each second will be

$$q_t = \frac{1}{6} Nm\overline{c_0}\,\overline{c_0^2}\left(\frac{\overline{c_2^2}}{\overline{c_0^2}} - 1\right).$$

Our assumption that the same law of distribution of velocities holds good both for the incident and the departing molecules, may be expressed as follows. If there is, among the incident molecules, a fraction $\frac{dN}{N}$ having velocities between c_0 and $c_0 + dc_0$, there will, among the departing molecules, be found the same fraction having velocities between c_2 and $c_2 + dc_2$, where $\frac{c_2}{c_0} = \frac{dc_2}{dc_0} = k_t$, k_t being a constant. This does not by any means imply that the same molecules which impinge on the body with the velocity c_0, must also depart with the velocity $k_t c_0$. The molecules departing with this latter velocity may very well be quite a different group to the one impinging on the body with the velocity c_0. Thus putting $c_2 = k_t c_0$ we have also $\overline{c_2} = k_t \overline{c_0}$ and $\overline{c_2^2} = k_t^2\,\overline{c_0^2}$. Substituting this in the expression for q_t, we get

$$q_t = \frac{1}{6} Nm\,\overline{c_0}\,\overline{c_0^2}\,(k_t^2 - 1)$$

or, since $p = \frac{1}{3} Nm\overline{c_0^2}$

$$q_t = \frac{1}{2} p\,\overline{c_0}\,(k_t^2 - 1).$$

If a molecule, in addition to its translational energy e_t, also possesses an internal energy e_i, e. g. rotational energy, which may be lost or changed by impact with a solid body, this internal energy must also be taken into account. c_p and c_v denoting the specific heat of the gas at constant pressure and constant volume respectively, we have

$$\frac{\overline{e_t}+\overline{e_i}}{\overline{e_t}} = \frac{\frac{2}{3}}{\frac{c_p}{c_v}-1}$$ from which $\overline{e_i} = \dfrac{\frac{5}{3}-\frac{c_p}{c_v}}{\frac{c_p}{c_v}-1}\cdot\overline{e_t}$. If we put

$f = \dfrac{\frac{5}{3}-\frac{c_p}{c_v}}{\frac{c_p}{c_v}-1}$ we get $\overline{e_i} = f\overline{e_t}$. We will now assume that e_i is distributed according to Maxwell's law, but entirely independent of e_t. Hence, since we have $\overline{e_t} = \frac{1}{2}m\overline{c_0^2}$ each molecule impinging on the body will, in losing its internal energy, give off an amount of heat whose mean value will be $\overline{e_2} = \frac{1}{2}m\overline{c_0^2}f$, and the number of impacts being $n = \frac{1}{4}N\overline{c_0}$, the whole amount of heat given off by the gas as originating from internal energy will be

$$\frac{1}{8}Nm\overline{c_0}\,\overline{c_0^2}f = \frac{3}{8}p\overline{c_0}\cdot f.$$

As the molecules leave the body and thus acquire internal energy, which we assume to be distributed according to Maxwell's law, each of them will in mean value carry with it an amount of energy proportional to e_i, and which may consequently be put equal to $\frac{1}{2}m\overline{c_0^2}k_i^2f$. Multiplying by the number of impacts $N = \frac{1}{4}N\overline{c_0}$, we get for the energy thus carried away by the gas from each square cm. in each second $\frac{1}{8}Nm\overline{c_0}\,\overline{c_0^2}k_i^2f = \frac{3}{8}p\overline{c_0}k_i^2f$. Deducting from this the amount of energy given off by the molecules until they had lost their internal energy, we get that the change in the internal energy of the molecules caused by the impacts will involve a total loss of heat for the body which will be, for each square cm. and each second

$$q_i = \frac{1}{8} N m \bar{c}_0 \overline{c_0^2} f(k_i^2 - 1) = \frac{3}{8} p \bar{c}_0 f(k_i^2 - 1).$$

The sum $q_t + q_i = q$ will be the total amount of heat carried away from each square cm. of the surface of the body in each second: accordingly it will be

$$q = \frac{1}{2} p \bar{c}_0 \left(k_t^2 - 1 + \frac{3}{4} f(k_i^2 - 1) \right).$$

If we can put $k_t = k_i = k$ here, we get

$$q = \frac{1}{2} p \bar{c}_0 \left(1 + \frac{3}{4} f \right) (k^2 - 1).$$

If we had assumed that $e_i = f e_t$ for each separate molecule, f would have to be substituted for $\frac{3}{4} f$ in this expression.

Imagining the possibility that k_t and k_i might be different, we attribute to the departing molecules a temperature T_t, taking only the translational energy into account, and another temperature T_i, taking only the internal energy into account. We put $\frac{T_t}{T_0} = \frac{\overline{c_2^2}}{\overline{c_0^2}} = k_t^2$ and similarly $\frac{T_i}{T_0} = k_i^2$. This may also be expressed by assuming that different coefficients of accommodation a_t and a_i hold good for the translational energy. In accordance with the definition we have $a_t = \frac{T_t - T_0}{T_1 - T_0}$ and $a_i = \frac{T_i - T_0}{T_1 - T_0}$ which gives the following relations between k and a

$$k_t^2 - 1 = a_t \frac{T_1 - T_0}{T_0} \quad \text{and} \quad k_i^2 - 1 = a_i \frac{T_1 - T_0}{T_0}.$$

Introducing this in the expression for q, we get

$$q = \frac{1}{2} p \bar{c}_0 \frac{T_1 - T_0}{T_0} \left(a_t + \frac{3}{4} f a_i \right).$$

Finally, introducing the value $\overline{c_0} = 14550 \sqrt{\frac{T_0}{M}}$ known from the kinetic theory, where M is the molecular weight of the gas, we get

$$q = 7275 \frac{p}{\sqrt{MT_0}} (T_1 - T_0) \left(a_t + \frac{3}{4} f a_i\right).$$

If we have $a_t = a_i = a$, we get

$$q = 7275 \frac{p}{\sqrt{MT_0}} (T_1 - T_0) \left(1 + \frac{3}{4} f\right) a,$$

where for a monatomic gas like helium we have $f = 0$. The expression thus found agrees with the one previously found by me for the molecular conduction of heat. If a_t and a_i are different, the latter expression may be employed if it is kept in mind that a is defined by

$$a = \frac{a_t + \frac{3}{4} f a_i}{1 + \frac{3}{4} f}.$$

On fixing the value of f for hydrogen it should, strictly speaking, be taken into account that the specific heat of hydrogen, and with it f, increases with increasing temperature, since the temperature for which we introduce f is the one at which the molecules depart from the heated body. In my experiments this temperature lies between 30° and 100° Celsius, the very range within which I have not been able to find good measurements of the temperature coefficient of the specific heat. This latter, however, will hardly exceed 0.0002, and if we entirely neglect it, the error introduced in f will presumably be less than 2 p. c.

and the error in a less than 0.7 p. c. We will therefore regard f as a constant within the range of temperatures here employed.

f might be found from values of C_v. Thus by M. Trautz and K. Hebbel[1] it has been found to be $C_v = 4.810$ kal./mol. grad at 16° C. If we put the gas constant $R = 1.986$, and keep in mind that we must here have $C_p - C_v = R$, we get $f = \frac{2}{3} \cdot \frac{C_v}{R} - 1$ and consequently $\frac{C_p}{C_v} = 1.4130$ and $f = 0.6146$. At 16° Scheel and Heuse[2] found $C_v = 4.875$ from which $\frac{C_p}{C_v} = 1.4074$ and $f = 0.6364$. Lummer and Pringsheim[3] found $\frac{C_p}{C_v} = 1.4084$, from which $C_v = 4.863$ and $f = 0.6325$. This latter value will be employed as it is in good accord with the previous one. If, instead, we had employed $f = 0.6146$, the values for a would only have been increased by 1 p. c., while $f = \frac{2}{3}$ would diminish the values found for a by 1.7 p. c. While thus the uncertainty of f does not essentially influence the determination of a, it would be different if we had reason to suppose that we must put $e_i = fe_t$ for each single molecule in calculating the amount of heat given off. If this were done, all the values for the coefficient of accommodation a found for hydrogen in heat conduction experiments would have to be multiplied by 0.942 or reduced by 5.8 p. c. Such a change would reduce the agreement found for hydrogen between $a' - a''$ and $a'_t - a''_t$, which would seem to indicate that we are justified in assuming that the internal energy e_i of the molecules is distributed quite independently of the distribution of the translational energy e_t.

[1] Ann. d. Phys. **74**, 1924, p. 320.
[2] Ann. d. Phys. **40**, 1913, p. 492.
[3] Wiedemann's Ann. **64**, 1898, p. 555.

If $f = 0.6325$, we get $1 + \frac{3}{4} f = 1.4744$ and the expression for q

$$q = 0.7372\, p\, \bar{c_0}\, (k^2 - 1)$$

$$q = 10726\, p\, (T_1 - T_0) \frac{1}{\sqrt{M T_0}}\, a.$$

From this expression for q we find the molecular conduction of heat by dividing by $p\,(T_1 - T_0)$. Hence it will be $10726 \frac{1}{\sqrt{M T_0}}\, a$ and with the stated definition of a must be anticipated to be independent of the temperature T_1 of the plate. If, on the other hand, a coefficient of accommodation a had been defined by the equation

$$a_1 = \frac{\sqrt{T_2} - \sqrt{T_0}}{\sqrt{T_1} - \sqrt{T_0}} = \frac{\bar{c_2} - \bar{c_0}}{\bar{c_1} - \bar{c_0}} = \frac{k - 1}{\frac{\bar{c_1}}{\bar{c_0}} - 1} = \frac{k - 1}{\sqrt{\frac{T_1}{T_0}} - 1}$$

the expression for q for small values $T_1 - T_0$ becomes

$$q = 10726\, p\, (T_1 - T_0) \frac{1}{\sqrt{M T_0}}\, a_1 \left(1 - (1 - a_1) \frac{1}{4\, T_0} (T_1 - T_0)\right).$$

Since a_1 can neither become 0 or 1, it will be seen tha with this definition of the coefficient of accommodation, q will be dependent on the temperature of the plate since the quantity $(1 - a) \frac{1}{4\, T_0}$ may be designated as a temperature coefficient.

For hydrogen we put $M = 2.016$, which gives

$$q = 7555\, p\, (T_1 - T_0) \frac{1}{\sqrt{T_0}}\, a,$$

which for $T = 293$ gives

$$q = 441.4\,p\,(T_1 - T^0)\,a \text{ for hydrogen, and}$$
$$q = 212.5\,p\,(T_1 - T_0)\,a \text{ for helium,}$$

since $M = 4.00$, $f = 0$ for this gas.

Calculation of the Molecular Radiometer Force.

We will now calculate the pressure with which the gas acts on the band mentioned in the previous part, applying the suppositions and terms made use of in the above.

We shall first calculate that part of the pressure which originates from the fact that the molecules lose their velocity c_0 upon impact with the band, and next the part originating from the fact that they leave the band with another velocity c_2.

A number of molecules $d^2n = \frac{1}{4\pi} dN \cdot c \cdot \cos x \cdot d\omega$ strikes each square cm. of the band each second coming from the solid angle $d\omega$ with velocities between c and $c + dc$. Each of these molecules, before its velocity has become 0, gives off a quantity of motion by the impact whose component in the directions of the surface normal is $mc \cdot \cos x$. If we calculate $d^2 nmc \cdot \cos x$, and integrate with respect to x and N, we get $\frac{1}{6} Nm\overline{c_0^2} = \frac{1}{2}p$. This was only what we might have anticipated, for the part of the pressure here calculated must be quite independent of whether or not there is equilibrium of temperature, and if there is equilibrium of temperature, half the pressure, that is, $\frac{p}{2}$, must originate from from the fact that the molecules lose their velocity, while the other half originates from the fact that they acquire a new velocity of equal mean value. Since the number of impacts $n = \frac{1}{4} N\overline{c_0}$, each of the molecules impinging on the band will thus make a contribution to the pressure whose mean value is $\frac{2}{3} m \frac{\overline{c_0^2}}{\overline{c_0}}$. This latter expression of course also holds good for the departing molecules if there is equilibrium of temperature, so if the velocities had been c_2 instead of c_0, each molecule would have contributed $\frac{2}{3} m \frac{\overline{c_2^2}}{\overline{c_2}}$ at its departure. In the case under consideration, however, the number of impacts is not $\frac{1}{4} N\overline{c_2}$ but $\frac{1}{4} N\overline{c_0}$, so for this part of the pressure we get $\frac{1}{6} Nm \frac{\overline{c_2^2}}{\overline{c_2}} \overline{c_0}$. As in the calculation of the loss of heat we put $c_2 = k_t c_0$, $\overline{c_2} = k_t \overline{c_0}$, $\overline{c_2^2} = k_t^2 \overline{c_0^2}$, which gives $\frac{1}{6} Nm \overline{c_0^2} k_t = \frac{1}{2} pk_t$. Adding this to the part of the pressure $\frac{1}{2} p$ found above, we get that the entire pressure p', with which the gas acts on the plate, is

$$p' = \frac{1}{2} p \, (k_t + 1).$$

The increase of pressure $p' - p$ originating from heating will thus be $p' - p = \frac{1}{2} p \, (k_t - 1)$, and hence $p' \, (p' - p) = \frac{1}{4} p^2 \, (k_t^2 - 1)$. For the loss of heat q, we found $q = \frac{1}{2} p c_0 \left(1 + \frac{3}{4} f\right) (k^2 - 1)$ so we have the following relation

$$\frac{q}{p' \, (p' - p)} = \frac{2 \, \overline{c_0}}{p} \left(1 + \frac{3}{4} f\right)$$

which is anticipated to be valid independently of the nature of the surface of the band.

In the expression found for p' we introduce the coefficient of accommodation a_t, as previously putting $k_t^2 - 1 = a_t' \frac{T_1 - T_0}{T_0}$ which gives

$$p' = \frac{1}{2}p\left(\sqrt{1 + a_t'\left(\frac{T_1}{T_0} - 1\right)} + 1\right).$$

Hence

$$p' - p = \frac{1}{2}p\left(\sqrt{1 + a_t'\left(\frac{T_1}{T_0} - 1\right)} - 1\right) \quad \text{and}$$

$$p'(p' - p) = \frac{1}{4}p^2 a_t' \frac{T_1 - T_0}{T_0}$$

and for small values of $T_1 - T_0$

$$p' - p = \frac{1}{4}a_t' \frac{T_1 - T_0}{T_0} p.$$

In the preceding part the coefficient of accommodation a_t was defined as the ratio between two temperature differences. If it had been defined as the ratio between two differences of molecular velocity, putting

$$a_{1,t} = \frac{\sqrt{T_2} - \sqrt{T_0}}{\sqrt{T_1} - \sqrt{T_0}} = \frac{k_t - 1}{\sqrt{\frac{T_1}{T_0}} - 1}$$

we should find

$$p' = p\left(1 + \frac{1}{2}a_{1,t}\left(\sqrt{\frac{T_1}{T_0}} - 1\right)\right) = p\left(1 + \frac{1}{2}a_{1,t}\frac{T_1 - T_0}{\sqrt{T_0}\left(\sqrt{T_1} + \sqrt{T_0}\right)}\right).$$

I have not attempted to measure directly the pressure p', but $p' - p''$, the difference of pressure between the two sides of the same band, one side of which was bright while the other was blackened, so that the two sides have different coefficients of accommodation a_t' and a_t''. The preceding equations give

$$p' - p'' = \frac{1}{2}p\left(k_t' - k_t''\right)$$

or

$$p'-p''=\frac{1}{2}p\left(\sqrt{1+a'_t\left(\frac{T_1}{T_0}-1\right)}-\sqrt{1+a''_t\left(\frac{T_1}{T_0}-1\right)}\right)$$

which may also be written

$$p'-p''=\frac{1}{2}p\,\frac{T_1-T_0}{T_0}\,\frac{1}{\sqrt{1+a'_t\left(\frac{T_1}{T_0}-1\right)}+\sqrt{1+a''_t\left(\frac{T_1}{T_0}-1\right)}}(a'_t-a''_t).$$

The quantity $p'-p''$ is designated as the radiometer pressure.

For small values of T_1-T_0 this will be approximately

$$p'-p''=\frac{1}{4}p\,\frac{T_1-T_0}{T_0}(a'_t-a''_t)\,\frac{1}{1+\frac{1}{4}(a'_t+a''_t)\,\frac{T_1-T_0}{T_0}}.$$

For $a'_t+a''_t=1$, which holds good approximately in my experiments, and for $T_0=293$ the expression will be

$$p'-p''=\frac{1}{4}p\,\frac{T_1-T_0}{T_0}(a'_t-a''_t)\,\frac{1}{1+0.000853\,(T_1-T_0)}.$$

If the second definition of the coefficient of accommodation had been applied and $a_{1,t}$ introduced, we should find

$$p'-p''=\frac{1}{2}p\,\frac{T_1-T_0}{T_0}\,\frac{1}{\sqrt{\frac{T_1}{T_0}}+1}(a'_{1,t}-a''_{1,t}).$$

This becomes approximately

$$p'-p''+\frac{1}{4}p\,\frac{T_1-T_0}{T_0}(a'_{1,t}-a''_{1,t})\,\frac{1}{1+\frac{1}{4}\,\frac{T_1-T_0}{T_0}}.$$

As will be seen, this is the same expression as we get from the above by putting $a'+a''=1$. It follows from this that my radiometer experiments cannot be expected to show which of the two definitions of the coefficient of accommodation is the best.

The preceding considerations only hold good when we can disregard the mutual collision of the molecules. Hence, if we wish to find a in a series of experiments at different pressures, we must find, by extrapolation, the value towards which a converges when p approximates to 0.

In order to settle whether the coefficient of accommodation a_i for the internal energy for hydrogen is equal to the coefficient of accommodation a_t for the translational energy, we determine $a' = \dfrac{a'_t + \frac{3}{4} f a'_i}{1 + \frac{3}{4} f}$ by the loss of heat from the blackened side of the plate, and the corresponding quantity a'' for the bright side of the plate. From this we find $a' - a''$. By measurement of the radiometer pressure we determine $a'_t - a''_t$, and if this equals $a' - a''$, we may anticipate that $a'_t = a'_i$ and $a''_t = a''_i$.

We may remark that the theoretical consideration leading to the expression for p' and hence for $p' - p''$, would be quite the same if we had assumed

1) that both sides of the plate had a surface of the same nature
2) that both sides of the plate were quite rough so that $a = 1$
3) that the two sides of the plate had different temperatures T'_1 and T''_1.

From this it will appear that, even if we measured T'_1 and T''_1, we should not arrive at any theoretical expression for $p' - p''$, if we had not $a = 1$ on both sides. This is not usually the case in the ordinary radiometers, in which the wings are double and one side is bright, the other blackened. By radiation we then get a difference in temperature between the two sides and in addition a difference between the coefficients of accommodation of the two sides. In this way the radiometer pressure becomes large, but it is a very complex phenomenon as temperature differences in the surrounding glass walls must also be taken into account.

In my experiments a simple calculation shows that the difference in temperature between the sides of the plate is so small that the radiometer pressure originating therefrom is smaller than the pressure originating from the emission of heat radiation, and this pressure is quite negligible compared with the radiometer pressures measured.

The Coefficient of Accommodation for single Impacts.

When a gas molecule impinges on the surface of a solid body, it is conceivable that the roughness of the surface will cause the gas molecule to enter in between the molecules of the solid body and rebound a number of times from them before again leaving the surface. It has been thus explained that a kind of adsorption takes place before the molecule is again released. As the exchange of energy between gas molecules, especially hydrogen molecules, and solid platinum has proved to be very imperfect, it must be presumed that the time during which the gas molecule remains in the adsorbed state is so short that it has only time to collide a few times with the platinum atoms. In other words, it is hardly in this case an adsorption effect, more probably we must suppose that some of the gas molecules get into the depressions in the surface of the platinum and rebound there some few times before leaving the surface.

Let us suppose that a platinum plate has a temperature such that the molecules of the surrounding gas would have the mean velocity $\overline{c_1}$ if there were equilibrium of temperature between the gas and the platinum. Let us suppose that a group of gas molecules which all have the velocity c_0 strike the platinum and rebound n times from the platinum surface before flying off with a velocity whose mean value will be denote by c_n. If $c_0 \lesseqgtr \overline{c_1}$ we must also have $c_n \lesseqgtr \overline{c_1}$, and we must have that the greater n becomes the more will c_n approximate to $\overline{c_1}$. It will be simplest to assume that for each addition impact of the molecules their velocities c_n will be augmented by a quantity dc_n which is proportional to the amount $\left(\overline{c_1} - c_n\right)$ lacking in the final velocities $\overline{c_1}$ reached by the molecules. This gives the equation

$$\frac{dc_n}{dn} = \left(\overline{c_1} - c_n\right) k,$$

where k is a constant valid for the gas and the solid body and independent of the roughness of the surface.

Integration from $n = 0$ to $n = n$ gives

$$\overline{c_1} - c_n = \left(\overline{c_1} - c_0\right) e^{-kn} \text{ from which } c_n - c_0 = \left(\overline{c_1} - c_0\right)(1 - e^{-kn}).$$

If the energy of the molecules had been introduced into the differential equation, the squares of the velocities being substituted for the velocities, the last equation would become

$$c_n^2 - c_0^2 = \left(\overline{c_1^2} - c_0^2\right)(1 - e^{-kn}).$$

Summing over all the velocities c_0, we get

$$\overline{c_n^2} - \overline{c^2} = \left(\overline{c_1^2} - \overline{c^2}\right)(1 - e^{-kn}).$$

We have, however, $\dfrac{\overline{c_n^2} - \overline{c_0^2}}{\overline{c_1^2} - \overline{c_0^2}} = \dfrac{T_n - T_0}{T_1 - T_0}$ and according to the definition, this quantity equals the coefficient of accommodation a, hence we have $a = 1 - e^{-kn}$ and $kn = -\log_e(1 - a)$. Consequently, if for a bright surface we have $a = a''$ and $n = n''$ and for a rough surface $a = a'$ and $n = n'$ we get

$$a' - a'' = e^{-kn''} - e^{-kn'} \quad \text{and} \quad \frac{n'}{n''} = \frac{\log(1 - a')}{\log(1 - a'')}.$$

This consideration thus gives the ratio $\dfrac{n'}{n''}$ determined by the coefficients of accommodation measured, but it gives us no direct information of the quantity n'', k being an unknown.

If, for a diatomic gas, we denote the coefficient of accommodation for the translational energy by a_t and that for the internal energy by a_i, and the corresponding values for k by k_t and k_i, and we have two surfaces of different degrees of roughness, the coefficients of accommodation a' and a'' determined by the conduction of heat from each of the surfaces will be given by

$$a' = \frac{a_t' + \frac{3}{4} f a_i'}{1 + \frac{3}{4} f} \quad \text{and} \quad a'' = \frac{a_t'' + \frac{3}{4} f a_i''}{1 + \frac{3}{4} f}.$$

If, for a diatomic gas, we put $f = \dfrac{2}{3}$, the equations will be

$$a' = \frac{1}{3}(2\,a_t' + a_i') \qquad a'' = \frac{1}{3}(2\,a_t'' + a_i'').$$

If a' and a'' have been determined by heat conduction experiments, we have the four unknowns a'_t, a''_t, a'_i and a''_i in the two equations. A third equation may be arrived at by determining $a'_t - a''_t$ by radiometer experiments, and, finally, a fourth equation expresses the fact that $\frac{n'}{n''}$ must have the same value for the translational and the internal energy, both these forms of energy being exchanged with the platinum at the same impacts. The fourth equation will be

$$\frac{\log(1 - a'_t)}{\log(1 - a''_t)} = \frac{\log(1 - a'_i)}{\log(1 - a''_i)}.$$

This latter equation will for the most part only be ap plicable in practice when all values of a differ so much from 1 that $1 - a$ may be determined with a reasonable percentage of accuracy, and this was not the case in my experiments.

Apparatus.

The platinum bands employed in making the measurements, five in all, were enclosed in two glass vessels which communicated with each other by means of a glass tube. From the latter a tube communicated with a pipette system and a pump. The tube was further provided with a trap kept cool in liquid air. Of the five platinum plates the two formed the sides of a torsion balance which was placed in one of the glass vessels. This consisted of a glass cylinder C (fig. 2) 26 cm. high and with an internal diameter of 23 cm. The edge of the cylinder was ground level and covered with a sheet of plate glass G (2 cm. thick) and fastened with picein to the cylinder so as to be airtight. A hole was bored in the middle of the glass plate over which was placed a tube R_1 25 cm. high and 3 cm. wide furnished with a plane-parallel window for the reading of the torsion balance. In other holes in the glass lid

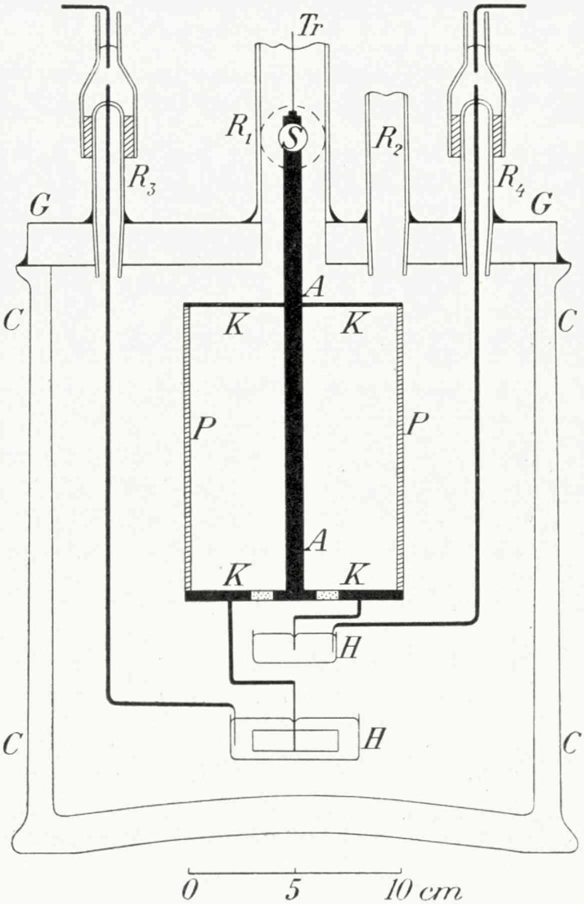

Fig. 2.

ground glass tubes could be inserted, one of which R_2 served to communicate with the other parts of the system, while two others R_3 and R_4 served as communication for the electric current to be sent through the platinum bands. The arrangement of the torsion balance will appear from fig. 2. The suspension wire *Tr* was of Wolfram. Its length was 11.97 cm. and its thickness was stated by the manufacturers to be 50 μ. The mirror S_1 was placed on a rod of aluminium A carrying two transverse rods of copper K. The lower transverse rod was, however, in isolated connection with the aluminium rod by intermediate pieces of mica. The platinum bands P were soldered to the ends of the copper rods. As shown in the figure, the conduction of the current takes place through two vessels with mercury H placed below, the current passing up through one band and down through the other. On account, amongst other things, of the rigidity of the mercury surfaces the torsion balance was made rather large. Thus the length of the platinum bands was c. 13.6 cm., their distance c. 10 cm. All dimensions were accurately measured, thus the length of one band was found to be 13.60 cm., its breadth 0.2488 cm., while the length of the other was 13.61 cm., its breadth 0.2569 cm. Hence the sum of the areas of the two plates (length $\times$ breadth) will be 6.88 ± 0.01 cm^2, so that a force of 1 dyn/cm^2 at right angles to the plates will give a torsion moment about the axis of 34.61 ± 0.07 dyn/cm. The platinum bands were of physically pure platinum from Heräus, their thickness was determined by weighing to be 0.00261 mm. A series of measurements of the electrical resistance of a band cut out of the same larger sheet as those mentioned here, gave the following temperature dependency $r_t = r_0\,(1 + \alpha t + \beta t^2)$ where $\alpha = 0.0039791$ and $\beta = -92.10^{-8}$. These constants for the temperature dependence of the resistance also apply to the other bands employed.

Before the apparatus was put together, the torsion moment of the suspension wire was determined without the aid of the two vessels of mercury. The period of oscillation was determined partly for the naked system, partly for the system with a convenient load so that its moment of enertia received a known increase. From this the moment of inertia of the naked system was found to be $I = 80.06 \pm 0.34$ g. cm.^{2} with the period of oscillation $\tau = 9.505 \pm 0.003$ seconds, and the moment of torsion D for the angle l, determined by $D = \pi^2 \frac{I}{\tau^2}$ was found to be $D = 8.246 \pm 0.047$ dyn cm/angle.

In the actuel measurements of the radiometer pressure the deflection of the torsion balance was measured by a reading telescope with a scale distance of 100.0 cm. A radiometer pressure of P dyn/cm.^{2} acting on the plates would thus cause a deflection of α cm. on the scale (reduced to angular measure where P and α are connected by the equation $P \cdot 34.61 = \frac{\alpha}{200} \cdot 8.246$ from which

$$P = \alpha \cdot (0.001264 \pm 0.000009),$$

where the uncertainty in the constant is thus c. 0.7 p. c.

The platinum bands of this apparatus were bright on one side and blackened with platinum black on the other. The layer of black was not very thick, lest the outer layer should have another temperature than the plate itself. Hence further blackening, e. g. with lamp black, was dispensed with. The bands were quite black to look at, and were of course placed in the torsion balance in such a way that increased pressure on the blackened sides would give torsion moments in the same direction for both plates.

With the apparatus here described, which we shall term the swinging apparatus in the following, the difference in pressure between the bright and the blackened sides of the bands, may, as will appear from the preceding part, be measured by deflection. This difference in pressure $p' - {}''p$ will in the following be termed the radiometer pressure. Further, by measuring the electrical resistance of the bands, their mean temperature may be found, and by measuring in addition the fall of potential the amount of electrical energy transformed to heat in the bands per second may be found, and this is again used to find the amount of heat given off to the surrounding gas per second from each cm.^{2} of the bands.

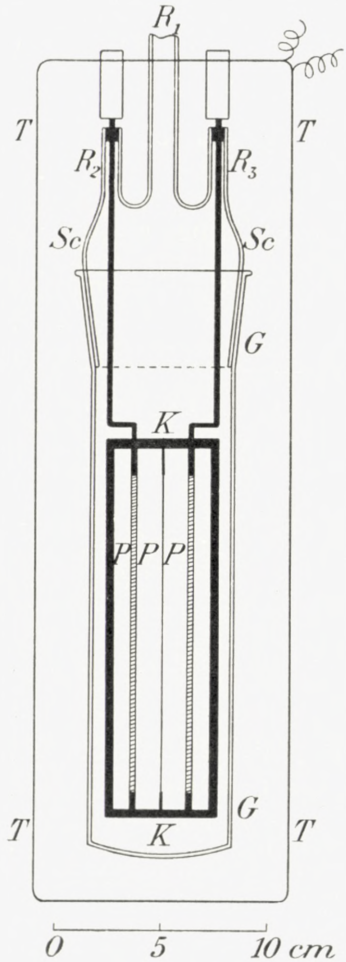

Fig. 3.

The remaining three of the five platinum bands were placed in a special glass container (fig. 3) consisting of a glass cylinder G closed at the bottom, 28 cm. long, and with an internal diameter of c. 6.5 cm. At the top the cylinder was closed so as to be airtight with a ground top-piece Sc with some narrower tubes. One of them R_1 communicated with the rest of the apparatus the other four (in the figure are only shown two of them R_2 and R_3) served to conduct an electrical current to the bands as shown in the figure. The bands P were extended in a frame of copper wire 5 mm. thick, and were all three soldered to the lower short side of the frame. At the top the bands were soldered to small pieces of metal which were isolated and carried through the upper short side of the frame. From these three pieces of metal and from the frame itself conducting wire was carried out through the apparatus so that measurements could be made for the band considered. The glass cylinder was placed in a somewhat larger glass vessel containing water, so that the temperature of the surroundings could be conveniently measured.

The band nearest one of the long sides of the frame had the surface turned forward, and was blackened with platinum black on both sides. The band nearest the other long side of the frame had also the surface turned forward and was bright on both sides. The band in the middle, had the edge turned forward and was blackened on one side and bright on the other. The object of turning the two other bands in the way described was as much as possible to prevent any action on the middle band as it is this band only which will serve to measure the radiometer pressure. If an electric current is passed through the band and the current is increased until the electrical resistance, and with it the mean temperature of the band, has reached the value r which is desired, and the current i_1 is measured, the temperature of the band and the heat it gives off may be found. The pressure on its blackened side being greater than that on its bright side, it will curve out, and if a microscope is focussed on the edge of the band near its middle, a deflection will be observed. The size of this deflection is not determined by the amount of the radiometer pressure only, but also by the tightening of the band and the increase of its length caused by heating. Hence measurement of the deflection is no good method of finding the radiometer pressure. Since, however, an electric current passes through the band, the strength of which may be accurately measured, a magnetic field is a well-suited means to restore the band to the position of equilibrium, and it is only necessary to know the requisite magnetic field in order to find the radiometer pressure. The magnetic field was produced by an electric current in a coil of wire consisting of 36 turns placed close together in two layers. Each turn approximated to a rectangle 40 cm. in height and 12 cm. in breadth, so that the turns could be just placed round the apparatus with the plane of the turns passing approximately through the platinum bands. By measuring the period of oscillation of a small magnet, and by calculation by means of the dimensions of the coil of wire, it was found with good agreement that the magnetic field produced by a current of 1 amp. in that place in the coil where the middle of the middle band is found, was 2.400 gauss. If the current in the band be i_1 and the current passed through the coil of wire i_2, in order to restore the band to the position of equilibrium, each cm. of the length of the band will be acted upon by the radiometer force $0.2400\, i_1\, i_2 \sin \alpha$, where α is the angle between the axis of the microscope, which coincides with the plane of the band, and the plane of the coil of wire. This angle was found to be 88.7°. If B is the breadth of the band, which was found by measurement to be 0.2484 $\pm$ 0.0003 cm., the radiometer pressure will be $P = \frac{0.2400}{B} \sin \alpha\, i_1\, i_2$ or substituting the numerical values $P = 0.9659\, i_1\, i_2$. The length of this band was 15.31 cm., the thickness 0.0002638. The constants for the bright band were: length 15.40 cm., breadth 0.2495 cm., thickness 0.2611, and for the band blackened on both sides: length 15.20 cm., breadth 0.2444 cm., thickness 0.0002632 cm.

Electrical Measurements. The conducting wires both in the fixed and the swinging systems were connected with cups of mercury outside the glass apparatus in order to enable the convenient changing of the measuring apparatus from one band to another. The band formed one side of a Thomson resistance bridge. Before the current was put on, the bridge was set to the resistance which the plate was to have, the resistance of the connecting wire being taken into account, and the strength of the current was now changed until the bridge galvanometer showed no deflection. In this way the current will not pass through the band any longer than necessary, and it will only be necessary to operate with some few, previously determined, temperatures of the band. The fall of potential in the platinum band was measured on a potentiometer, and was thus compared with a Weston standard cell kept at a constant temperature. For in spite of the small temperature coefficient of this cell, it had proved rather sensitive to variations in temperature. Although much care was thus taken and the usual precautions were observed with regard to thermo-forces etc., it turned out that the accuracy in the measurement of the heat given off by the band was only just what must be strictly required. This is due to the fact that the radiation of heat from the band in many cases constitutes by far the greater part of the loss of heat. Current reversers were placed in the system, so that measurement was always made with currents in opposite directions. This is of special importance in measuring the radiometer force in order to eliminate the effect of the earth's magnetic field which, though it was in great part compensated for by a permanent magnet, will still act on the current in the band. The strength of the current i_2 passed through the coil to compensate for the radiometer force in the central band was measured by a precision ammeter.

The object in using two different methods of measuring the radiometer pressure was partly to ensure verification, and partly to employ two different degrees of sensitivity. For the swinging system is much more sensitive than the fixed band, but on the other hand it is unsuited for measuring such large radiometer pressures that the scale passes outside the field of vision in the telescope. A fairly high degree of sensitivity was chosen for the swinging system precisely with the object of rendering possible measurement at low pressures, and a change of sensitivity by the insertion of a thicker suspension wire was out of question as, owing to the giving off of adsorbed gas, the whole apparatus must be left for some months with vacuum before it could be worked at such low pressures as a few bars. Within the range in which the radiometer pressure approaches its maximum, heating of the bands of the oscillating system to c. 5° will bring the scale out of the field of vision. Hence the range of measurement of the swinging system was increased in the following way.

Two large coils of wire were placed close by the large glass cylinder with their ends directed towards the oscillating system. An electric current i_2 which could be measured by a precision ammeter could be passed through the coils. The current produces a magnetic field proportional to i_2, and if i_1 is the current in the band, this will be influenced by a force proportional to $i_1 i_2$. The measuring was done as follows. First the position of equilibrium with $i_1 = 0$ and $i_2 = 0$ was determined. A current was then passed through the bands which were thus heated and influenced by the radiometer force. The current is increased to i_1', which is so large that the deflection a just does not exceed the scale. A current i_2' was then passed through the coil by which the swinging system is influenced by a moment $k i_1' i_2'$, and i_2' is so adjusted as to produce a deflection b, in the opposite direction of the deflection a, and likewise as large as possible. The turning moment originating from the magnetic force, which is proportional to $i_1' i_2'$ has thus produced the deflection $a+b$, and as might be anticipated, a constant ratio k between $a+b$ and $i_1' i_2'$ has been found. If now the current i_1 required to obtain the desired temperature is passed through the bands, the magnetising current i_2 will be thus adiusted that a suitable small deflection c will be obtained. The turning moment produced by the radiometer force with the current i_1 will thus be determined by the fact that it would have given a deflection $c + k i_1 i_2$ if i_2 had been 0.

The period of oscillation for the swinging system was c. 20 seconds. Suitable damping had been provided, one platinum wire, the end of which was immersed in mercury, being furnished with a wing of platinum. This wing was completely immersed in the mercury. Thus through the surfaces of the mercury there passed only the 2 mm. thick platinum wires which were well centred in the system so as to break the surface as little as possible when swinging. That the surface of the mercury would possess a certain rigidity was to be anticipated, hence no position of equilibrium or deflection was ever determined with the system at rest, but only while it was performing oscillations of which averages were taken in a suitable way. It turned out that the adjustment was more accurate when the experimenter tapped the plank on which the swinging apparatus was placed. This will generate waves in the mercury surfaces so that the solid crust is broken.

The platinum bands in the swinging system had a total resistance which at 0° was $r_0 = 4.0508$ ohms. The resistance in the connecting wires (mainly inside the glass cylinder) was measured to be 0.026 ohms. Account was taken of this latter resistance in the adjustment to Thomson's bridge, so that if, for instance, a mean temperature of the platinum bands of 101.6° C. is desired, the adjustment to Thomson's bridge, should be $r = 5.676$ ohms. Consequently the development of heat per second in both platinum bands will be $\left(\frac{V}{r}\right)^2 (r - 0.026)$ where V is the fall of potential over the resistance r read on the potentiometer.

As an example of a measurement made with the swinging system at the pressure $p = 0$, that is to say after the mercury diffusion pump had been at work for a long time, we quote the following figures from the journal

Hour	r	t_1	t_0	i_1	V	U	H
14^h 55	5.676	101.6	17.50	0.317	1.7770	23.15	29.30
15^h 00	5.676	101.6	17.52	−0.317	1.7763	40.50	29.00

The measurements at the hours 14^h 55 and 15^h 00 were made with the same strength of current in opposite directions. The temperature of the bands was to be $t_1 = 101.6°$ so Thomson's bridge was adjusted to 5.676. t_0 is the temperature of the glass vessel in which the swinging system was placed. i_1 is the current through the bands measured by a precision ammeter, but this quantity is only used for verification as it is not accurate enough. On account of the insertion of the ammeter in the system the figures given under i_1 must be multiplied by 100/101 to give the current through the bands. Thus in the case given the current determined by the ammeter is 0.314, while the mean value of $\frac{V}{r}$ is 0.31301, which is used. Under V the potentiometer readings are tabulated and under U the magnitude ot the deflection readings is given. The deflections themselves must be reckoned from the positions of rest given under H. It will at once be apparent that currents in opposite directions do not give equal deflections in opposite directions. One is -6.15, the other 11.50, mean value 2.68, which gives a radiometer pressure P of 0.0034 bars. This shows that the apparatus is not quite exhausted, for the radiation pressure alone can only cause a deflection which is about 200 times smaller. Even if liquid air is placed somewhere on the apparatus, this cannot remove the mercury vapours from the container since the two mercury-cups are inside it. This is of course a disadvantage, but I have not been able to avoid it. The pressure is such that the deflection would be produced by a hydrogen pressure of 0.12 bars. It will immediately be seen that the apparatus may be used for the measurement of pressures, but that such a type requires adjusting. From r and V the electric development of heat in the bands is found to be 0.5536 watt, part of which is lost by heat conduction through the ends of the bands. This part is calculated from a formula which I have previously given[1] and is stated to be 0.0361 watt. If this is divided by the area of the bands (length $\times$ breadth), we get that the amount of heat given off per second from each cm^2 of one side of the bands plus that given off from each cm^2 of the other side is 0.07522 watt/cm^2 = 752200 erg./(sec. cm^2).

[1] Ann. d. Phys. Bd. 34, 1911, S. 628.

In the great majority of measurements made with the swinging system magnetic compensation was utilised and converted into deflections in the way mentioned. For each of the pressures p used, given in bars, and for each of the temperatures of the bands used, the radiometer pressure $p' - p''$ is given also in bars, and likewise, the loss of heat given in Ergs/cm^2 sec. From the latter is subtracted the amount of heat $Q_{p=0}$ given off through the sides of the bands in vacuum (lowest pressure obtainable) and the remainder will be the amount of heat $q' + q''$ lost by conduction through the gas per second from 1 cm^2 of the one side of the band plus 1 cm^2 of the other side of the band.

In a similar way we find the amounts of heat given off to the surrounding gas from the fixed bands, among which the measurements for the band which is bright on both sides are of special interest, since we may take it for granted that the molecular conduction of heat from this band is the same as that from the bright side of the bands blackened on one side only.

Measurements at low Pressures.

Within a range of 0 to 20 bars three series of experiments with hydrogen, I, II, and III, were carried out. In one of these series, at each of the pressures employed, measurements with various temperatures of the bands were made immediately after one another. The lowest temperature of the band, was taken first, then the higher ones, whereupon the series was repeated in the reverse order. In this way each result becomes the mean of two repetitions. The temperature of the surroundings was about 20°.

In this series of experiments the loss of heat from the bands of the swinging system was determined, further the loss of heat from the band blackened on both sides, and from the band which was bright on both sides. In addition the radiometer force of the swinging system was measured.

The losses of heat were corrected for loss by conduction through the ends, and for radiation by means of the measurements made in vacuum. The result was then divided by the difference in temperature $T_1 - T_0$ between the band and its surroundings, and by the hydrogen pressure p, further by the areas of the surfaces of the bands. The quantity thus found is an approximate value of the molecular heat conducting power of the hydrogen and is designated $\frac{1}{p}\frac{q'}{T_1 - T_0}$ for the blackened side of the bands of the swinging system, and $\frac{1}{p}\frac{q''}{T_1 - T_0}$ for the bright side. The molecular conductivity for hydrogen-band is defined as the value towards which $\frac{q}{p(T_1 - T_0)}$ converges when p and $T_1 - T_0$ approximate to 0.

Thus for the swinging system $\frac{q' + q''}{p(T_1 - T_0)}$ is determined by measuring the loss of heat from the bands. On the supposition that the surface of the band which is bright on both sides is of the same nature as that of the bright sides of the bands in the swinging system, $\frac{q''}{p(T_1 - T_0)}$ will be determined by the loss of heat from the bright band. From the values thus found for $\frac{q' + q''}{p(T_1 - T_0)}$ and for $\frac{q''}{p(T_1 - T_0)}$, $\frac{q'}{p(T_1 - T_0)}$ and $\frac{q' - q''}{p(T_1 - T_0)}$ may be found. These quantities are given in the table below (series I, hydrogen), while the loss of heat from the band blackened on both sides has been tabulated but not used in the calculation. The measurements with this band have only been employed to check results. As a matter of fact, it turned out that for this band $\frac{q'}{p(T_1 - T_0)}$ was but little different from $\frac{q'}{p(T_1 - T_0)}$

for the blackened sides of the bands of the swinging system.

The results of the measurements of the radiometer force with the swinging system were divided by the area of the bands, only one side being included in the area. This results in a quantity $p'-p''$ which may naturally be designated as the radiometer pressure and is conceived as the difference between the pressure p' of the hydrogen on the blackened sides of the bands and its pressure p'' on the bright sides. If the radiometer pressure be divided by $p(T_1-T_0)$, an almost constant value will appear. With decreasing values of p and T_1-T_0 this quantity will converge towards a value characteristic of the gas and the band which may be termed the "molecular radiometer pressure". In the table below the values of $\frac{p'-p''}{p(T_1-T_0)}$ calculated from the observations are given. Since the values for $\frac{q'+q''}{p(T_1-T_0)}$, for $\frac{q''}{p(T_1-T_0)}$ and for $\frac{p'-p''}{p(T_1-T_0)}$ found from the observations vary but slightly with the temperature difference T_1-T_0, only the approximate values for this quantity are given in the table. In the tabulated values the unit erg has been used to give the amount of heat, and the unit bar for pressure.

The measurements with helium were treated in essentially the same way.

Tables.

Series I (Hydrogen).

p	T_1-T_0	$\frac{q'+q''}{p(T_1-T_0)}$	$\frac{q'}{p(T_1-T_0)}$	$\frac{q''}{p(T_1-T_0)}$	$\frac{(p'-p'')10^6}{p(T_1-T_0)}$	$a'+a''$	$a'-a''$	$a'_t-a''_t$
3.035	20	453	330	136	—	1.036	0.408	—
	40	453	327	132	—	1.039	0.423	—
	60	431	322	134	—	0.992	0.365	—
	80	444	307	135	—	1.024	0.391	—
5.041	20	470	—	—	299	1.085	—	0.379
	40	458	—	—	310	1.059	—	0.395
	60	457	—	—	306	1.060	—	0.392
	80	460	—	—	307	1.071	—	0.396
8.327	20	451	—	—	329	1.048	—	0.432
	40	446	—	—	331	1.039	—	0.437
	60	449	—	—	327	1.051	—	0.435
	80	455	—	—	327	1.077	—	0.437
10.80	20	459	299	134	314	1.077	0.424	0.428
	40	450	297	132	321	1.060	0.415	0.440
	60	467	297	132	308	1.104	0.458	0.425
	80	444	297	131	302	1.053	0.407	0.419
15.93	20	418	293	123	292	0.998	0.384	0.423
	40	435	292	121	298	1.041	0.434	0.434
	60	448	285	120	292	1.074	0.465	0.417
	80	423	276	120	285	1.026	0.414	0.420
20.29	20	436	292	124	261	1.057	0.417	0.398
	40	432	289	123	267	1.048	0.413	0.410
	60	430	287	122	261	1.047	0.410	0.402
	80	426	283	122	259	1.041	0.403	0.401

Series II (Hydrogen).

p	$\frac{a'+a''}{p\,(T_1-T_0)}$	$\frac{(p'-p'')\,10^6}{p\,(T_1-T_0)}$	$a'+a''$	$a'_t-a''_t$
5.105	449	321	1.046	0.413
10.19	443	308	1.052	0.423
15.25	435	285	1.051	0.416
20.29	425	269	1.046	0.416
		Mean values . . .	1.049	0.417

Series III (Hydrogen).

p	$\frac{a'+a''}{p\,(T_1-T_0)}$	$\frac{(p'-p'')\,10^6}{p\,(T_1-T_0)}$	$a'+a''$	$a'_t-a''_t$
0.925	451	324	1.035	0.394
1.846	459	321	1.055	0.396
2.736	468	322	1.077	0.401
3.677	461	322	1.064	0.404
4.587	461	317	1.066	0.403
5.493	459	311	1.063	0.400
6.396	456	306	1.057	0.396
7.295	455	302	1.056	0.395
		Mean values . . .	1.059	0.399

Series I (Helium).

p	T_1-T_0	$\frac{q'+q''}{p(T_1-T_0)}$	$\frac{q'}{p(T_1-T_0)}$	$\frac{q''}{p(T_1-T_0)}$	$\frac{(p'-p'')10^7}{p(T_1-T_0)}$	$a'+a''$	$a'-a''$	$a'_t-a''_t$
5.205	30	284.5	189.9	85.3	4050	1.408	0.564	0.519
	60	254.8	183.7	82.1	3839	1.285	0.456	0.510
	80	256.5	181.8	81.0	3768	1.310	0.482	0.512
	110	254.0	182.8	79.7	3618	1.322	0.491	0.510
	130	251.2	182.1	78.8	3549	1.323	0.490	0.511
10.355	30	262.2	184.2	80.7	3894	1.336	0.516	0.5219
	60	251.1	178.4	77.9	3713	1.304	0.496	0.5164
	80	253.5	176.2	76.7	3609	1.333	0.526	0.5139
	110	246.1	175.3	75.9	3502	1.318	0.504	0.5163
	130	244.0	174.4	75.3	3420	1.323	0.505	0.5158
15.403	30	244.8	180.5	78.7	3578	1.282	0.460	0.5005
	60	240.0	174.9	76.3	3476	1.281	0.468	0.5046
	80	237.8	172.5	75.5	3408	1.285	0.471	0.5065
	110	237.6	170.6	74.5	3292	1.308	0.489	0.5066
	130	236.8	170.1	73.9	3224	1.320	0.496	0.5076
20.304	30	241.4	177.7	77.8	3487	1.298	0.466	0.5076
	60	235.5	172.4	74.8	3378	1.290	0.473	0.5105
	80	234.4	169.9	73.8	3312	1.300	0.484	0.5123
	110	233.7	168.3	72.8	3206	1.320	0.498	0.5134
	130	233.0	167.3	72.3	3131	1.333	0.506	0.5140
25.103	30	246.1	174.3	75.8	3420	1.356	0.526	0.5167
	60	235.3	169.1	73.2	3283	1.321	0.503	0.5146
	80	232.4	167.0	71.5	3200	1.321	0.511	0.5136
	110	231.6	165.4	71.3	3089	1.341	0.518	0.5133
	130	230.2	164.8	71.1	3030	1.349	0.517	0.5150

Series II (Helium). $T_1 - T_0 = 80°$.

p	$\frac{q'+q''}{p(T_1-T_0)}$	$\frac{q'}{p(T_1-T_0)}$	$\frac{q''}{p(T_1-T_0)}$	$\frac{(p'-p'')10^7}{p(T_1-T_0)}$	$a'+a''$	$a'-a''$	$a'_t-a''_t$
5.303	250.8	177.2	73.5	3765	1.265	0.533	0.5128
10.584	247.9	175.3	73.8	3610	1.273	0.537	0.5152
15.843	240.8	173.1	74.1	3477	1.257	0.510	0.5190
21.080	238.5	170.5	73.6	3313	1.265	0.538	0.5159
26.296	234.9	168.1	73.3	3180	1.267	0.532	0.5157
31.491	232.2	165.6	73.0	3047	1.272	0.539	0.5136

Series III (Helium). $T_1 - T_0 = 80°$.

p	$\frac{q'+q''}{p(T_1-T_0)}$	$\frac{q'}{p(T_1-T_0)}$	$\frac{q''}{p(T_1-T_0)}$	$\frac{(p'-p'')10^7}{p(T_1-T_0)}$	$a'+a''$	$a'-a''$	$a'_t-a''_t$
1.016	209.7	181.0	73.3	3534	—	—	—
2.027	235.3	180.1	74.7	3847	—	—	—
3.034	240.6	179.7	75.2	3785	1.193	0.445	0.5048
4.038	241.0	182.4	76.8	3793	1.194	0.431	0.5109
5.037	247.8	181.5	76.6	3834	1.228	0.466	0.5208
6.032	247.2	179.7	76.6	3756	1.225	0.464	0.5146
7.023	246.9	181.0	76.8	3739	1.224	0.461	0.5169
8.011	246.5	179.2	76.6	3708	1.222	0.461	0.5170
8.995	245.5	179.1	76.6	3682	1.216	0.454	0.5180
9.976	246.1	180.7	76.7	3636	1.219	0.457	0.5159

From the observed values for $\frac{q'+q''}{p(T_1-T_0)}$, $a'+a''$ has been calculated, and q'' has likewise been employed for the calculation of a'', and from the values thus found for $a'+a''$ and for a'', $a'-a''$ has been calculated and tabulated. Thus from the two columns $a'+a''$ and, $a'-a''$ the coefficients of accommodation a' for the blackened side and a'' for the bright side determined in each separate experiment, can be found. From the observed values for $\frac{p'-p''}{p(T_1-T_0)}$, $a'_t-a''_t$ was calculated, that is to say, the difference between the coefficients of accommodation for the translational energy of the molecules for the two sides of the plates. In the calculations the following interpolation formulas have been employed, the constants being found by the method of least squares.

Hydrogen: For the bright band

$$a'' = \frac{\sqrt{T_0}}{7555}\left(1-0.000274\,(T_1-T_0)\right)^{-1}(1-0.00577\,p)^{-1}\cdot\frac{q''}{p(T_1-T_0)} = 0.315.$$

For the swinging system

$$a'+a'' = \frac{\sqrt{T_0}}{7555}\left(1-0.000150\,(T_1-T_0)\right)^{-1}(1-0.00306\,p)^{-1}\cdot\frac{q'+q''}{p(T_1-T_0)} = 1.050.$$

$$a'_t-a''_t = 4\,T_0\left(1+0.000358\,(T_1-T_0)\right)(1+0.01445\,p)\,\frac{p'-p''}{p(T_1-T_0)} = 0.415.$$

Helium: For the bright band

$$a'' = \frac{\sqrt{T_0}}{3637.5}\left(1+0.000701\,(T_1-T_0)\right)(1+0.00558\,p)\,\frac{q''}{p(T_1-T_0)} = 0.411.$$

For the swinging system

$$a'+a'' = \frac{\sqrt{T_0}}{3637.5}\left(1+0.000652\,(T_1-T_0)\right)(1+0.00587\,p)\,\frac{q'+q''}{p(T_1-T_0)} = 1.320.$$

$$a'_t - a''_t =$$
$$4\,T_0\left(1 + 0.001300\,(T_1 - T_0)\right)(1 + 0.009523\,p)\,\frac{p' - p''}{p\,(T_1 - T_0)}$$
$$= 0.5120.$$

That a slight formal difference in the formulas for hydrogen and helium has been used is of no importance. Thus the values extrapolated for $p = 0$ and $T - T_0 = 0$ will be for hydrogen $a' + a'' = 0.420$ and $a'' = 0.315$ from which we find $a' = 0.735$ and $a' - a'' = 0.420$ while the radiometer force gives $a'_t - a''_t = 0.415$.

These two differences in the coefficients of accommodation differ so little that they must be regarded as equal. The difference found is so small that it would disappear entirely if instead of 0.315 we had found $a'' = 0.318$, and we cannot expect an accuracy of 1 p. c. in this and the other quantities. From the table we find that the mean deviation between the values for $a' - a''$ and $a'_t - a''_t$ calculated separately from the observations is 0.024, which gives a mean deviation of the means amounting to 0.007.

In a similar way was found for helium $a' + a'' = 1.320$ and $a'' = 0.411$ from which we find $a' = 0.909$ and $a' - a'' = 0.498$ while the radiometer force gives $a'_t - a''_t = 0.512$.

In the experimental series II and III, measurements were only made at a single temperature of the bands, viz. c. 373° Kelvin. The observed values of $\frac{q' + q''}{p\,(T_1 - T_0)}$ and $\frac{p' - p''}{p\,(T_1 - T_0)}$ and the values for $a' + a''$ and $a'_t + a''_t$ calculated from them are given in the tables. In this calculation the temperature coefficients given in the interpolation formulas were employed.

These series exhibit satisfactory agreement and show that even with such small variations in pressure as 0.9 bars, which have been employed in series III, rather exact measurements of the coefficients of accommodation can be made, both by the heat loss method and the radiometer method. This could not be known at the outset when we consider that a more or less constant evaporation takes place from the mercury in the oscillating system.

If we compare the separate values in each series of coefficients of accommodation, we shall observe that the variations in $a' - a''$ are greater than in the series for $a_t' - a_t''$. Thus the radiometer force measurements can be executed with considerably greater percentage accuracy than the heat loss measurements. This is due to the fact that, in the latter measurements, we have a vacuum correction mainly from radiation which is many times larger than the difference to be measured. Thus the error caused in this way is not outweighed by the fact than in measuring the heat loss we have the exact electrical methods of measurement at disposal. That the values for $a' - a''$ have been found so very nearly equal to $a_t' - a_t''$ for helium would seem to indicate that the theoretical considerations applied are correct, and that measurements of this kind may be made with sufficient accuracy. This affords a certain guarantee that, from the agreement between $a' - a''$ and $a_t' - a_t''$ for hydrogen, we may infer that the coefficient of accommodation for the internal energy of the molecules is of the same magnitude as the coefficient for the translational energy.

Radiometer Force at Higher Pressures.

The radiometer force at high pressures has recently been subjected to investigation by several authors, partly theoretically and partly experimentally. Thus we may mention W. H. Westphal, E. Einstein, A. Einstein, H. E. Marsh, E. Condon, L. B. Loeb, G. Hettner, J. Mattauch, A. Sternthal, P. S. Epstein, M. Czerny, Irma Bleibaum, Th. Sexl, E. Brücke and W. Littwin. A fairly full list of the literature is found e. g. in a work by Theodor Sexl[1].

From the above mentioned works it appears that at high pressures the radiometer force must be regarded as a edge effect on the plate or band, and that it may be put proportional to $p\lambda^2 \Delta T/T$ where p is the pressure, λ the mean free path of the surrounding gas, T its absolute temperature, and ΔT the difference in temperature between the two sides of the radiometer plate. Since $p\lambda = \lambda_1$, where λ_1, denotes the mean free path at a pressure of 1 bar, the expression may also be written $\lambda_1^2 \frac{\Delta T}{T} \cdot \frac{1}{p}$, that is to say that, with the same pressure, the radiometer force increases with λ_1^2 for different gases, while for the same gas it varies inversely as the pressure. Since, in addition, it is known that at low pressures the radiometer force varies directly as the pressure, these demands have been combined in a single expression, in which the radiometer force is made proportional to $\frac{ap}{1+bp^2}$. If the radiometer force is plotted against $\log p$ a symmetrical curve will appear, as shown by Westphal. That this agrees with the observations with fairly good approximation will appear from various works.

The results thus found by others have been confirmed in my experiments, hence I have applied them in calculating the results of my experiments. It would seem, however, that there is some deviation, if but slight, from the above-mentioned symmetry, and moreover it proved neces-

[1] Zeitschrift für Physik. Bd. 52, p. 249, 1928.

sary to assume a more complex dependency of temperature than that assumed by merely putting the radiometer pressure $p' - p''$ proportional to the temperature difference $T_1 - T_0$ between the plate and its surroundings.

Fig. 4 is typical of all the measurements made; $\log_{10} p$ is given as the abscissa. As ordinates are given the quan-

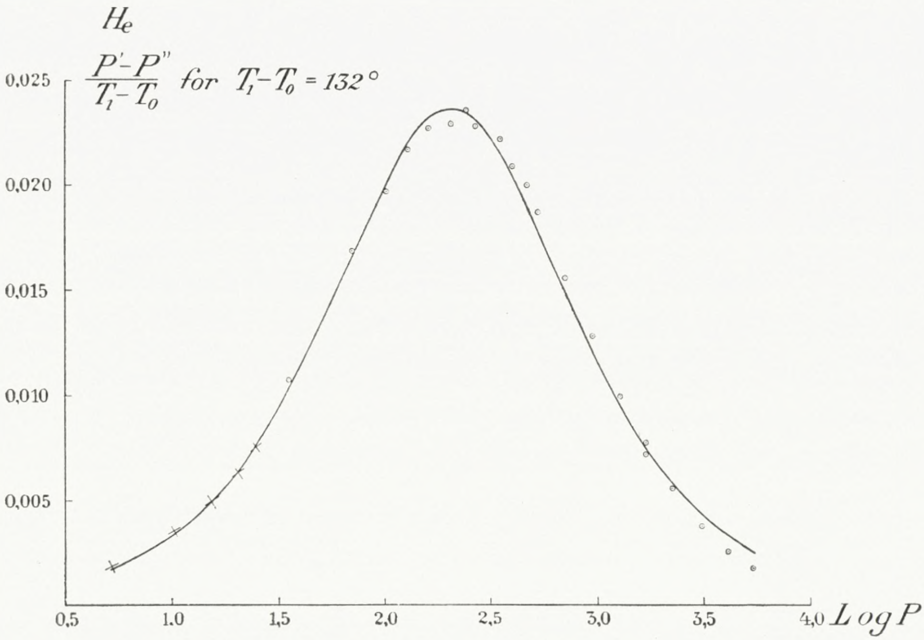

Fig. 4.

tity $\frac{p' - p''}{T_1 - T_0}$ for helium, $T_1 - T_0$ having the value 132° ($T_0 = 293°$ abs.). The ordinates marked with dots originate from measurements with the central band in the fixed system, the ordinates marked with crosses from measurements with the swinging system. The curve represents a symmetrical function which pretty nearly satisfies all the observations. The constants of the function have been determined by the method of least squarest with the exclusive use of measurements made with the central band.

Despite this it will be seen that the crosses marking the observations from the swinging system very nearly lie on the curve. At rather high pressures, however, systematic differences between the observations and the above-mentioned calculated function-values occur, and at the highest pressures the forces measured are decidedly smaller than those calculated.

The formula applied is as follows

$$p' - p'' = \frac{\frac{1}{2} p \left(\sqrt{\frac{T_1}{T_0}} - 1\right) (a'_t - a''_t)}{1 + b_1 p + c_1 p^2}$$

or

$$\frac{p' - p''}{T_1 - T_0} = \frac{a'_t - a''_t}{2 T_0 \left(\sqrt{\frac{T_1}{T_0}} + 1\right)} \cdot \frac{p}{1 + b_1 p + c_1 p^2}$$

with the following values of the constants

for helium	$a'_t - a''_t = 0.4898$	$b_1 = 0.005450$	$\sqrt{c_1} = 0.005234$
for hydrogen	0.363	0.00819	$0.00744.$

These constants vary very slightly only with the temperature difference $T_1 - T_0$. They decrease somewhat when $T_1 - T_0$ increases, and thus we may put $a'_t - a''_t = 0.4898$ $(1 - 0.0005\,(T_1 - T_0))$ for helium and use the very same temperature-dependency for hydrogen and for the two other constants b_1 and $\sqrt{c_1}$.

The table below gives the values of the radiometer pressure per degree $\frac{p' - p''}{T_1 - T_0}$ observed at various pressures p and temperature differences $T_1 - T_0$. It further gives the quantity Δ to be added to the observed value in order to obtain the value determined by the formula.

Helium.

Observed values for $10^4 \dfrac{p'-p''}{T_1-T_0}$ and for $10^4 \varDelta$.

	$p =$	35.7	68.9	99.7	128.2	163.5	204.1	245	271
$T_1-T_0 =$	32°	133	205	221	250	259	261	268	253
$10^4 \varDelta$		−16	−20	+1	−8	−6	−6	−17	−5
	62°	117	185	218	227	244	245	250	239
		−4	−7	−3	+9	+3	+5	−4	+3
	82°	115	180	211	227	238	239	247	235
		−5	−5	0	+5	+5	+7	−4	+4
	112°	111	174	204	224	232	234	241	231
		−4	−4	+1	+2	+6	+7	−3	+3
	132°	108	169	199	219	228	231	237	228
		−3	−3	+3	+3	+6	+7	−2	+3

	$p =$	357	395	463	530	712	956	1283	1701
$T_1-T_0 =$	32°	244	223	218	194	160	132	97	85
$10^4 \varDelta$		−17	−5	−16	−6	−4	−6	+3	−7
	62°	230	212	205	186	152	122	93	72
		−7	+2	−6	−1	+2	+2	+6	+5
	82°	227	211	203	186	153	123	95	72
		−7	+1	−6	−2	0	0	+3	+5
	112°	225	207	202	186	155	127	96	74
		−7	+2	−8	−5	−4	−5	+1	+2
	132°	223	210	202	188	156	129	101	77
		−8	−4	−9	−9	−7	−8	−4	−1

Helium.

Observed values for $10^4 \frac{p'-p''}{T_1-T_0}$ and for $10^4 \Delta$.

	$p =$ 1722	2283	3064	4113	5521
$T_1 - T_0 = 32°$	75	65	53	41	32
$10^4 \Delta$	+ 3	— 5	— 7	— 6	— 5
62°	70	53	37	26	19
	+ 7	+ 7	+ 8	+ 9	+ 8
82°	71	52	37	25	18
	+ 5	+ 7	+ 8	+ 10	+ 8
112°	73	53	37	25	17
	+ 3	+ 6	+ 8	+ 9	+ 9
132°	74	57	38	26	18
	+ 2	+ 2	+ 7	+ 8	+ 8

Hydrogen.

Observed values for $10^4 \frac{p'-p''}{T_1-T_0}$ and for $10^4 \Delta$.

	$p =$ 30.67	55.29	99.68	179.7	323.9	583.9
$T_1 - T_0 = 30°$	72.9	98.3	131.8	122.6	99.3	61.1
$10^4 \Delta$	— 2.6	+ 3.9	— 4.9	+ 5.3	+ 5.0	+ 11.3
60°	70.2	100.2	126.1	125.3	101.8	65.8
	— 2.2	— 1.1	— 2.5	0.0	+ 0.9	+ 5.7
80°	67.8	98.0	122.5	125.0	101.9	67.2
	— 1.3	— 0.9	— 0.9	— 1.3	— 0.2	+ 3.8
130°	62.4	91.6	116.8	122.7	103.2	66.8
	+ 0.6	+ 1.0	+ 0.1	— 2.7	— 3.6	+ 3.1

For small values of $T_1 - T_0$ the equation becomes

$$\frac{p' - p''}{T_1 - T_0} = \frac{1}{4\,T_0}\,(a_t' - a_t'')\,\frac{p}{1 + b_1 p + c_1 p^2}$$

but it must be remarked that with the values given above for b_1 and c_1 the equation is valid only for the breadth of band $B = 0.2484$ cm. used by my experiments, and the measurements do not permit to decide with certainty how B enters into the constant b_1.

When the pressure p decreases the formula converges against the theoretical expression. The radiometer force is a surface effect and the quantity $\frac{p' - p''}{T_1 - T_0}$ is independent of B.

For very great values of p the equation gives

$$\frac{p' - p''}{T_1 - T_0} = \frac{1}{4\,T_0}\,(a_t' - a_t'')\,\frac{1}{c_1 p}.$$

As A. Einstein and others have shown and as I have been able to verify the radiometer force at high pressures is an edge-effect. The force $(p' - p'')\,B$ acting of each unit of length of the band must consequently be independent of B. From this follows that c_1 must be proportional to B. If the thickness of the band could be considered infinite small against its breadth B and this again infinite small against the length of the band and its distance from the walls of the vessel, then B would be the only length characterising the apparatus and the unit for this length could be taken only from the property of the gas. As a length characterising the gas could be taken $\lambda_1 = p\lambda$ the mean free path at the pressure 1 Bar. Putting $\lambda_1 = 19.76$ for helium and $\lambda_1 = 12.42$ for hydrogen and putting $c_1 = cB/\lambda_1$ we get for helium $\sqrt{c} = 0.0467$ and for hydrogen $\sqrt{c} = 0.0526$

$$\frac{p'-p''}{T_1-T_0} = \frac{1}{4\,T_0}\,(a_t'-a_t'')\,\frac{p}{1+b_1 p+c\frac{B}{\lambda_1}p^2} =$$

$$\frac{1}{4\,T_0}\,(a_t'-a_t'')\,\frac{1}{\sqrt{c\frac{B}{\lambda_1}}}\,\frac{1}{\frac{1}{p\sqrt{\frac{cB}{\lambda_1}}}+p\sqrt{\frac{cB}{\lambda_1}}+\frac{b_1}{\sqrt{\frac{cB}{\lambda_1}}}}.$$

From this expression the symmetrical shape is seen at once and also that the radiometer pressure becomes maximum when $p = p_m$ where $p_m\sqrt{\frac{cB}{\lambda_1}} = 1$ or when the mean free path $\lambda = \lambda_m$ where $\lambda_m = \sqrt{cB\lambda_1}$. Thus by inserting the experimental values for c_1 we find for helium $p_m = 191$ Bar, $\lambda_m = 0.1034$ cm. and for hydrogen $p_m = 134$ Bar and $\lambda_m = 0.0924$ cm. From this may be expected that the maximum value of the radiometer force occurs at a pressure inverse proportional to the square root of the breadth of the band and at a mean free path direct proportional to the same quantity.

The maximum value of the radiometer force is determined by

$$\left(\frac{p'-p''}{T_1-T_0}\right)_{\max} = \frac{1}{4\,T_0}\,(a_t'-a_t'')\,\frac{1}{2\sqrt{c\frac{B}{\lambda_1}}+b_1}.$$

With the experimental values for $\sqrt{c}$ and b_1 we find $2\sqrt{c\frac{B}{\lambda_1}}+b_1$ equal to 0.01592 for helium and to 0.02307 for hydrogen giving for $\left(\frac{p'-p''}{T_1-T_0}\right)_{\max}$ equal to 0.02625 for helium and to 0.01342 for hydrogen. This latter gas gives thus only half the radiometer pressure of that found in helium.

It was found by my experiments with fairly good approximation that $b_1 = \sqrt{c_1} = \sqrt{\frac{cB}{\lambda_1}}$ what gives

$$\frac{p'-p''}{T_1-T_0} = \frac{1}{4\,T_0}\,(a_t'-a_t'')\,\frac{1}{\sqrt{\frac{cB}{\lambda_1}}}\,\frac{1}{1+\frac{1}{p\sqrt{\frac{cB}{\lambda_1}}}+p\sqrt{\frac{cB}{\lambda_1}}}$$

and

$$\left(\frac{p'-p''}{T_1-T_0}\right)_{\max} = \frac{1}{12\,T_0}\,\frac{a_t'-a_t''}{\sqrt{\frac{cB}{\lambda_1}}}\,.$$

That the maximum radiometer pressure should be inverse proportional to $\sqrt{B}$ and that the maximum radiometer force on the unit of length of the band should be directly proportional to $\sqrt{B}$ has been tried by some experiments to be mentioned later on. The trial has not been fully in favour of these theoretical considerations. It must be confessed that the experiments are unsufficient to form an equation which should be able to hold good for all values of B and λ_1.

It must also be born in mind that the breadth of band by my experiments cannot be regarded as vanishing in comparison to the distance from the walls of the surrounding vessel. This distance ought also to appear in an equation demanding completeness. I have not investigated how it enters but the following consideration might perhaps give some guidance.

Let $A'A''$ (fig. 5) be a diameter of the glass cylinder surrounding the plate B which is blackened on the side turned towards A'. The blackening on one side has the same effect as if the plate had a surface of the same kind on both sides, but a higher temperature on the blackened than on the bright side. This difference in temperature gives rise to horizontal currents in the gas, the gas being drawn from the cold side of the plate round the edge to the warm side. The plate acts with a certain force on the gas, and the gas reacts on the plate with a reactionary force which is, precisely, the radiometer force (at high pressures). The radiometer force may also be explained by the fact that the gas on the blackened side of the plate has a higher temperature and therefore a higher pressure than on the bright side. This difference in pressure, which is the radiometer pressure, is very small since the gas from the blackened side escapes along the glass wall to the bright side of the plate. The greater the resistance to this gaseous current, the greater will be the radiometer force which must be anticipated when all other conditions are equal. The smaller the radius chosen for the glass cylinder, the greater must be the resistance to the gaseous currents to be anticipated, and consequently also the greater the radiometer force. Hence it is to be anticipated that with a formula of the type here applied, the radius of the glass cylinder must be introduced into the denominator.

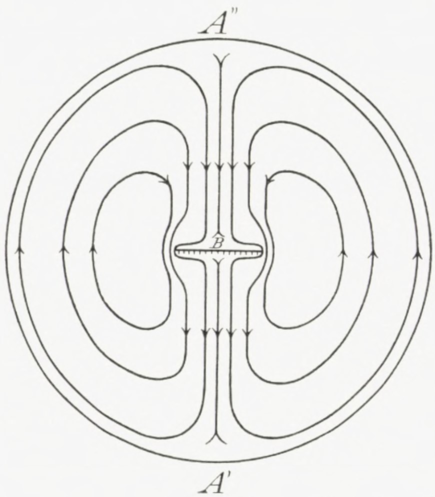

Fig. 5.

It may be anticipated that there must be a certain relation between the radiometer pressure $p' - p''$ and the losses of heat q' and q'' per cm^2 per second from the two sides of the band. The loss of heat from all three bands has therefore been measured at the pressures and differences of temperature at which measurements of $p' - p''$ have been made.

The results of these measurements are contained in the following tables. For the sake of convenience the quantities

$$Q'' = \frac{1}{100} \cdot \frac{q''}{T_1 - T_0} \text{ Erg/Grad for the bright band}$$

$$Q' = \frac{1}{100} \cdot \frac{q'}{T_1 - T_0} \text{ Erg/Grad for the black band}$$

$$Q''' = \frac{1}{100} \cdot \frac{q'''}{T_1 - T_0} \text{ Erg/Grad for both sides of the middle}$$

band have been tabulated:

Hydrogen.

$T_1 - T_0$	$p =$	30.7	55.3	99.7	179.7	324	584	1290	3670	7190	14390
30°	Q''	33.6	58.4	94.5	154.4	234.2	326.1	464	573	619	644
	Q'	78.0	127.9	202.0	300.8	404.9	490.6	598	648	666	673
	Q'''	111.1	185.5	307.7	450.5	626	789		1129	1189	1225
60°	Q''	33.2	56.4	93.2	151.4	230.8	322.3		581		
	Q'	76.8	126.9	200.4	298.5	408.6	496.2		674		
	Q'''	107.6	180.6	295.8	441.2	618	783		1152		
80°	Q''	33.1	56.1	92.5	150.2	228.9	319.3	463			
	Q'	76.1	125.3	198.8	297.9	406.2	500.6	621			
	Q'''	107.1	178.1	293.2	439.4	616	785	1005			
130°	Q''	32.6	56.9	91.3	148.0	226.0	318.5				
	Q'	71.4	123.9	197.1		408.6	506.7				
	Q'''	104.6	176.7	288.6	435.0	614	792				

Helium.

$T_1 - T_0$	$p =$	35.7	68.9	99.7	128.2	163.5	204.1	271
30°	Q''	25.5	47.2	66.6	82.7	101.6	121.3	148.1
	Q'	58.9	105.9	142.9	170.4	202.0	232.9	272.6
	Q'''	79.2	146.7	200.1	243.1	291.3	337.9	404.9
60°	Q''	25.1	46.2	65.5	81.5	100.1	119.6	148.7
	Q'	57.8	103.0	139.6	167.5	199.4	229.2	269.1
	Q'''	78.0	144.3	197.0	240.1	287.2	333.2	401.7
80°	Q''	25.0	46.4	65.2	81.2	99.8	119.2	147.9
	Q'	56.8	102.2	138.1	165.7	197.2	227.3	268.3
	Q'''	77.4	144.4	195.8	238.5	286.2	332.1	399.9
110°	Q''	25.0	46.4	65.1	80.9	99.3	118.6	147.1
	Q'	56.4	101.5	137.0	165.1	196.4	226.5	267.2
	Q'''	75.7	142.5	194.3	237.3	283.0	328.1	397.7
130°	Q''	25.0	46.3	65.0	80.7	99.1	118.4	146.9
	Q'	56.4	101.0	136.5	164.3	195.7	226.3	266.9
	Q'''	76.8	142.3	193.6	236.4	281.6	328.9	393.9

$T_1 - T_0$	$p =$	245	357	463	395	530	712	956
30°	Q''	143.7	187.9	223.6	191.8	232.2	274.8	317.9
	Q'	262.3	320.7	358.7		372.1	415.3	448.5
	Q'''	391.8	475.5	543.3	500.7	572.7	643.0	725.5
60°	Q''	139.5	183.3	218.5	191.0	231.1	274.4	316.5
	Q'	257.2	315.4	355.4		372.6	417.9	451.0
	Q'''	377.1	473.3	539.9	493.0	571.7	646.7	724.0
80°	Q''	138.2	181.0	215.9	190.1	232.1	274.5	317.2
	Q'	256.0	314.5	355.1		373.1	419.5	458.0
	Q'''	375.2	471.8	539.0	492.2	572.3	649.9	728.5

Helium.

T_1-T_0	$p=$	245	357	463	395	530	712	956
110°	Q''	136.6	179.8	213.9	190.2	230.7	274.8	318.3
	Q'	255.0	313.2	355.9		375.4	423.5	464.0
	Q'''	372.2	469.6	537.3	491.2	573.3	651.8	734.4
130°	Q''	135.7	178.2	212.8	189.9	230.8	275.1	319.0
	Q'	254.3	313.5	356.5		376.2	424.0	463.5
	Q'''	369.8	467.0	536.9	489.9	574.3	655.3	738.7

T_1-T_0	$p=$	1283	1722	1701	2283	3064	4113	5521
30°	Q''	358.2	400.3	406.9	439	472	495	511
	Q'	482.5	520.5	511.0	533	549	563	575
	Q'''	787.4	863.1	834.8	885	923	952	983
60°	Q''	358.7	402.9	405.8	441	477	501	523
	Q'	492.5	528.5	520.0	545	563	577	604
	Q'''	790.4	855.3	845.3	898	939	975	1005
80°	Q''	360.3	403.3	406.1	444	479	507	528
	Q'	495.0	530.0	525.0	552	570	589	597
	Q'''	794.9	861.3	851.4	903	950	986	1017
110°	Q''	362.6	405.3	407.0	447	484	511	537
	Q'	504.5	539.5	534.5	561	582	598	611
	Q'''	798.1	870.4	862.9	918	965	1004	1039
130°	Q''	364.0	408.9	408.3	449	489	516	542
	Q'	504.5	543.0	541.5	568	590	605	623
	Q'''	805.9	876.8	868.8	925	979	1019	1054

A comparison between the observed values of the radiometer forces and the heat losses did not lead to any simple result so we shall not discuss this matter further. We shall merely mention that the loss of heat for each of the bands may be expressed approximately by the formula

$$q = q_0 \frac{a + cp}{1 + cp} \frac{p}{1 + bp} (T_1 - T_0).$$

Form very small values of p this formula gives $\frac{q}{p(T_1 - T_0)} = q_0 a$ which is the molecular conductivity of heat, a denoting the coefficient of accomodation. As is well known, at high pressures the loss of heat is independent of a. This does, in fact, agree with the measurements and is expressed in the formula by the factor $\frac{a + cp}{1 + cp}$ which approaches 1 with increasing p whatever be the value of a. For large values of p the formula gives

$$q = \frac{q_0}{b} (T_1 - T_0)$$

where the quantity $\frac{q_0}{b}$ is determined by the dimensions of the apparatus and the heat conductivity $\varkappa$ of the gas. The measurements with hydrogen gave $\varkappa = 0.000429$ cal./(Grad. cm. Sek.) which agrees well with previous determinations of this quantity.

Radiometer force produced by Radiation.

In order to enable the experimental results to be treated with greater facility and certainty the experimental series hitherto mentioned have been carried out in such a way that the experiments of each separate series were performed at a constant temperature difference $T_1 - T_0$ between the band and its surroundings. In the usual radiometer measurements it is not, however, this quantity which is kept constant, but the amount of heat supplied to the band per second, the band being usually heated by radiation. In that case the amount of heat S supplied to the band by radiation per square cm. per second in the stationary state equals the sum $S_0 + q$ of the amount of heat S_0 given off from the band by radiation and the amount q lost by conduction through the gas, both quantities reckoned per cm.^{2} and per second. Both S_0 and q have been measured in the experiments, hence at any pressure at which measurements have been made we can find the value of $T_1 - T_0$ with a given value of $S_0 + q = S$. Loss of heat by conduction through the ends is here neglected.

By way of example we may assume that the value of S is so small that S may be put proportional to $T_1 - T_0$ as a sufficient approximation to the law of radiation. The measurements with the central band then show that we may put

$$S_0 = 5800\ (T_1 - T_0)\ \mathrm{erg/(sec.\ cm^2)}.$$

If by way of example we put $S = S_0 = 100000$, we get that $T_1 - T_0 = 17.2°$ in vacuum, where $q = 0$. From the measurements with helium we find, for instance with $p = 35.7$ bars, that $q = 7680\ (T_1 - T_0)$, so that, consequently, we have

$$S - (5800 + 7680)\ (T_1 - T_0), \text{ from which we find}$$

$$T_1 - T_0 = \frac{S}{13480}.$$

If again $S = 100000$, we get $T_1 - T_0 = 7.4°$. It will thus be seen that even a very small helium pressure will reduce the temperature of the band from $T_1 - T_0 = 17.2°$ in vacuum to 7.4°. In the table below $T_1 - T_0$ has thus been calculated for various helium pressures p, and in the third line the radiometer force $p'-p''$ corresponding to $T_1 - T_0$ has been tabulated. The supply of heat $S = 100000$ erg (sec. cm²) in helium

$p =$	0	10.4	35.7	68.9	128.2	357	712	5521
$T_1 - T_0 =$	17.2	11.8	7.4	5.0	3.4	1.9	1.4	0.9
$p'-p'' =$	0	0.0415	0.0796	0.0843	0.0742	0.0424	0.0219	0.0016

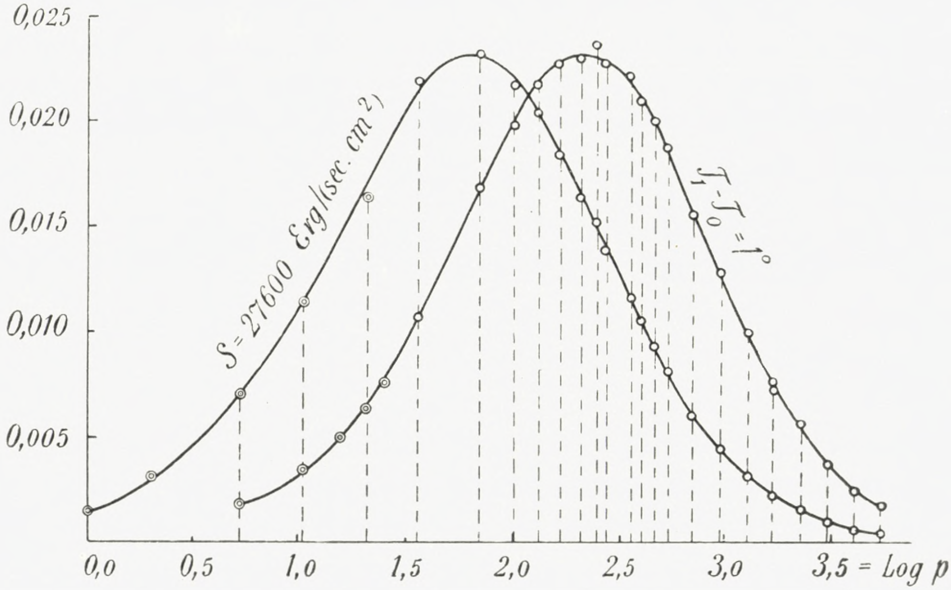

Fig. 6.

From this table it will be seen what a great influence the heat conductivity of the gas has on the temperature of the band and thus on the radiometer pressure. The table gives only a brief extract of the measurements, whereas the results of the measurements at all pressures are given in fig. 6. In the figure log p is plotted against the radiometer force $p'-p''$ given in bars. The curve on the right applies to the difference in temperature $T_1 - T_0 = 1°$ calculated from the observations, in which $T_1 - T_0$ was c. 130°, by dividing the radiometer forces observed by the observed difference in temperature. The observed values are thus identical with those marked in fig. 4, but the curve drawn has been determined by graphical smoothing and is thus somewhat different from the curve in fig. 4 which shows the course of the symmetrical function. By measurement one may easily convince oneself that the graphically smoothed curve is not quite symmetrical.

For each point of observation of the curve for $T_1 - T_0 = 1^\circ$, a value for $p' - p''$ has been calculated on the supposition that the heat supply S had had the constant value 27600 ergs/(cm^2 sec.). This value has been chosen such that the two curves get the same maximum value for $p' - p''$ and a common scale of ordinates. Corresponding points have been connected by dotted ordinates. It will be seen that the curve for $S = 27600$ ergs/(cm^2 sec.) determined by graphical smoothing has also a fairly symmetrical course, though the symmetry is less pronounced than in the curve for $T_1 - T_0 = 1^\circ$. It will especially be observed that while the radiometer force for $T_1 - T_0 =$ constant has its maximum at $p =$ c. 200 bars, this maximum is attained at $p =$ c. 60 bars, i. e. at a much lower pressure, when the supply of heat is kept constant.

In all measurements hitherto mentioned the radiometer force is exclusively due to the fact that the band has a different coefficient of accommodation on its two sides, since we may take it for granted that the two sides have the same temperature. If a large radiometer force is desired, however, it will be of advantage to arrange the experiment in such a way that the blackened side of the plate will get a higher temperature than the bright side. In order to examine how much the radiometer force might be increased in this way the following experiments were made.

A torsion balance (fig. 7) was made and suspended by a wolfram wire in a cylindrical glass tube. To the vertical rod were attached two horizontal bars between which the two platinum bands *Pt* were extended. One band was single, the other double, the latter consisting of two sheets, one covering the other, arranged in such a way that they were kept a slight distance apart by thin glass threads. One of these sheets was bright on both sides, the other was blackened with platinum black on the side turning outward. The single band was blackened, on one side, bright on the other. Each of the sides of the bands could be heated by radiation from an incandescent lamp and the deflection of the torsion balance was measured by a microscope with ocular scale. It is obvious that when the single band is heated by radiation, its two sides will get practically the same temperature, so that the radiometer force of this band will be due exclusively to the difference in the coefficient of accommodation on the two sides. If, however, the double band is heated by radiation, the side which is heated will get a higher temperature than the opposite side.

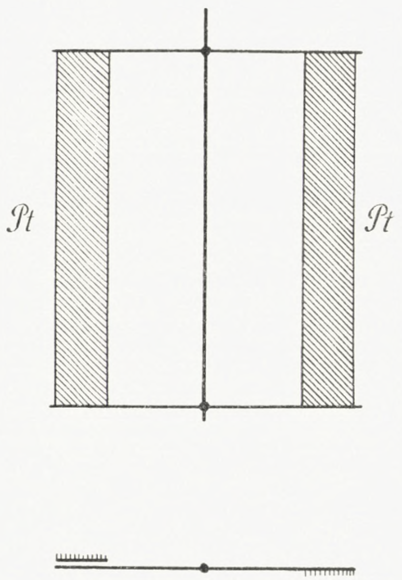

Fig. 7.

The length of the bands was c. 4.5 cm., their breadth

c. 3 mm., their thickness 0.0026 mm., and the distance between their outer edges c. 3.5 cm.

A series of measurements were made in which each side of one band was irradiated by a 10 candle power lamp at a distance of 30 cm. from the band. The measurements were made at a series of different pressures p, the magnitude of which was not measured, but which approximate roughly to a quotient series. In the figures below the deflections measured, and thus the radiometer force, are plotted against $\log p$. The ordinates and hence the radiometer forces in all the curves are directly comparable. On the extreme left of the figures is marked the radiometer force at the lowest pressure obtainable. This is marked on the axis of the abscissa with $p = 0$, though the radiometer force shows that there has been a very appreciable quantity of gas left in the apparatus, during these measurements. The apparatus had not been left with vacuum for any very long time, and liquid gas was not employed.

From fig. 8, illustrating measurements in hydrogen, it will be seen that the lower curve, resulting from radiation on the blackened side of the single band, is almost symmetrical and takes the same course as the curve in fig. 6 which applies to helium $S = 27600$ ergs/(sec. cm^2), and against which the numerical values for p and $p' - p''$ are marked. This might be expected at the outset. From the upper curve, resulting from the radiation on the blackened sides of the double band it will be seen that, at low pressures, the radiometer force is about 4 times as great as by radiation on the single band. This is chiefly due to the fact that there is a very appreciable difference in temperature between the two parts of the double band owing to the small conduction of heat between the bands at low pressures. Hence the effect is as if the bright side of the double band had a coefficient of accommodation which was nearly zero. We might also say that there is a difference in temperature between the two sides of the double band which likewise produces a radiometer force which is added to that originating from the difference in the coeffici-

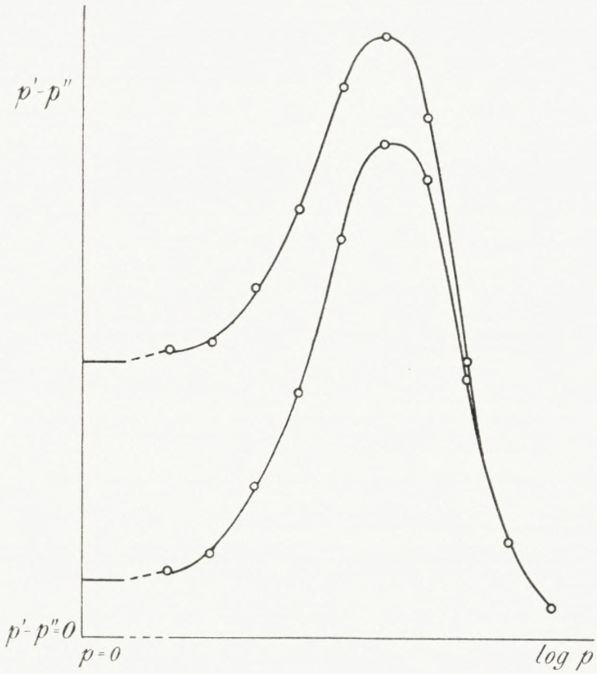

Fig. 8. Radiometer force in hydrogen due to radiation on the black side of the double band gives upper curve of the single band gives lower curve.

ents of accommodation on the two sides of the single band. To this must be added that it must be anticipated that the part of the double band, one side of which has been blackened will get a somewhat higher temperature than the single band, the loss of heat by radiation being somewhat less from the double than from the single band. It will appear from the figures that the difference between the radiometer force $p' - p''$ of the double and the single band decreases with increasing pressure p. The ratio between the maximum values is 1.23, and at the highest pressures p the radiometer force on the single band is as great as that of the double band. In this and the following figures the positive ordinates denote that the radiometer force has the same direction as the rays of light.

The lower curve in fig. 9 shows the radiometer force with radiation on the bright side of the single band. The

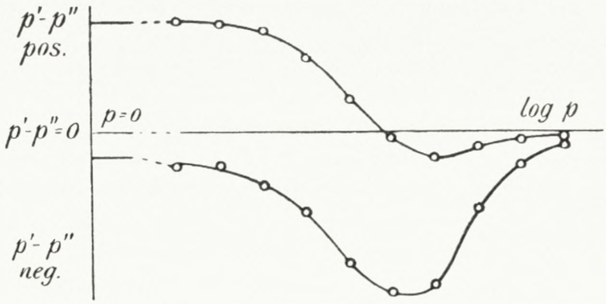

Fig. 9. Radiometer force in hydrogen due to radiation on the bright side
of the double band gives upper curve
of the single band gives lower curve.

radiometer force is negative at all pressures caused by the absence of any appreciable difference in temperature between the sides of the band, and by the fact that the radiometer force is only due to the difference in the coefficients of accommodation of the two sides of the band. Hence the radiometer force takes the same direction with radiation on the bright and on the black side. In the former case the difference in temperature between the band and its surroundings, and with it the radiometer force, will be 3 times as low at any pressure as in the latter case.

The upper curve in fig. 9 shows the radiometer force with radiation on the bright side of the double band. In this case, too, the radiometer force is 3 times as low at low pressures as with radiation on the black side. But with increasing pressure the radiometer force diminishes and becomes negative at high pressures, despite the fact that the bright side of the band of course at any pressure has a higher temperature than the black side. This is

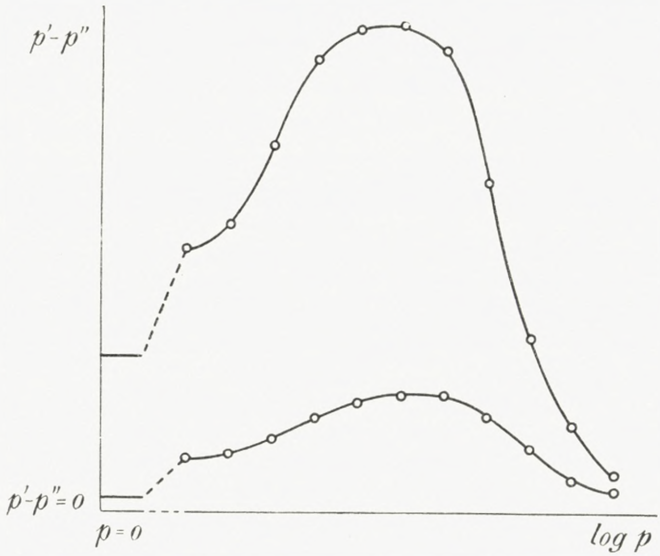

Fig. 10. Radiometer force in atmospheric air due to radiation on the black side
of the double band gives upper curve
of the single band gives lower curve.

simply explained by the great heat conductivity of hydrogen at high pressures, which will cause the temperature of the black band to become so much higher than that of the gas that the radiometer pressure will be greater on the black than on the bright side.

In order to compare the radiometer force in atmospheric air with that in hydrogen the measurements shown in figs. 10 and 11 were made. From fig. 10 it will be seen that with radiation on the black side the radiometer force on the double band is about 5 times as great at low pressures as with the same radiation on the single band. The maximum value of the radiometer force is about 4 times as great for the double as for the single band, and the maximum value for the double band in atmospheric air is about equal to the maximum value for the single band in hydrogen. In the case of the double band the maximum radiometer force is only increased by about 25 p. c. on changing from atmospheric air to hydrogen.

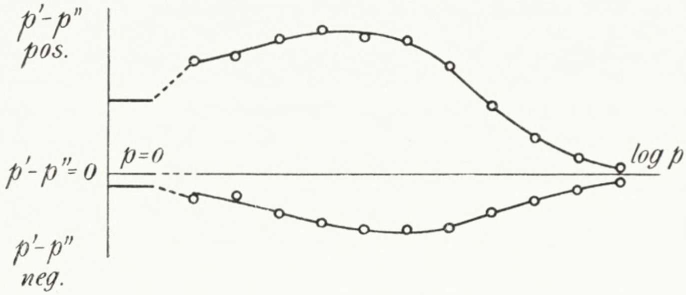

Fig. 11. Radiometer force in atmospheric air due to radiation on the bright side
of the double band gives upper curve
of the single band gives lower curve.

On comparing the upper curves in figs. 9 and 11 it will be seen that they take very different courses. In atmospheric air the radiometer force will not become negative for any value of p with radiation on the bright side of the double band.

In order to obtain some information as to the influence of the breadth of the band on the radiometer force, some preliminary experiments were made with the torsion balance shown in fig. 7. The double band was removed and replaced by a single one, which was bright on one side and blackened on the other. This new band was half the breadth of the other band of the torsion balance.

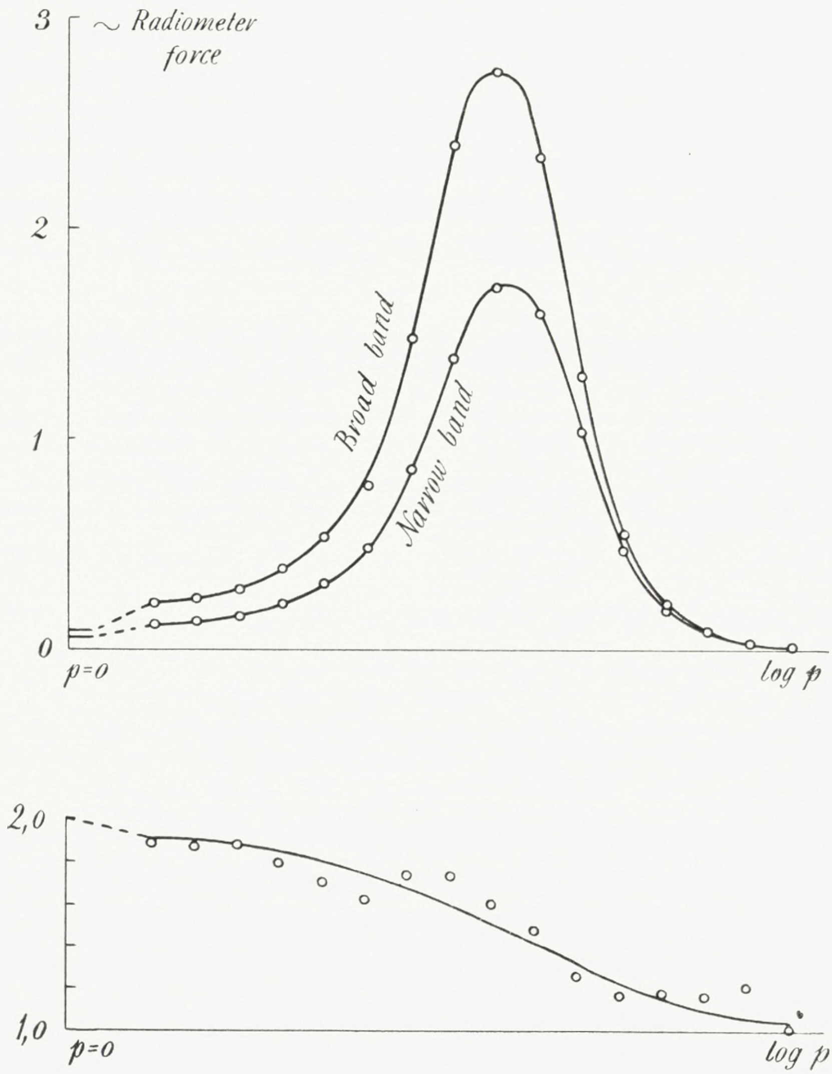

Fig. 12.

Some experiments with hydrogen were made. The experimental results have been shown in the usual way in the upper part of fig. 12. The initial pressure, shown to the right in the figure, was 33.5 mm. mercury pressure, and between two successive measurements the pressure was reduced to c. $\frac{1}{2.6}$ of its value.

As will appear from the figure, the effect is the same on the narrow and the broad band at high pressures. Hence the radiometer force is independent of the breadth of the band, the effect being an edge effect. With diminishing pressures the radiometer force becomes greater on the broad than on the narrow band, and at very low pressures the ratio converges towards 2. The force is a surface effect, the radiometer force being directly proportional to the breadth of the band.

It seems to judge from the figure that the maximum value of the radiometer force occurs at a pressure which is between 1.4 and 1.6 times as high for the narrow as for the broad band.

In the lower part of the figure the ratio between the radiometer force on the broad and on the narrow band is indicated. At very high pressures, on the extreme right in the figure, this ratio is very uncertain, but measurements other than those given here show that, with increasing pressure, the ratio converges towards 1, and that, on the whole, the ratio increases with decreasing pressure. In the vicinity of the pressures at which the radiometer force attains its maximum the ratio seems to increase most. To judge from the figure, it seems to be 1.6 rather than $\sqrt{2} = 1.41$.

A somewhat more detailed investigation of the radiometer forces near the maximum values was made, the pressure being reduced at small intervals. The pressure was reduced each time to $\frac{1}{1.19}$ of its value. The results of the measurements are given in fig. 13 in the same way as in fig. 12. From the figure the maximum value of the

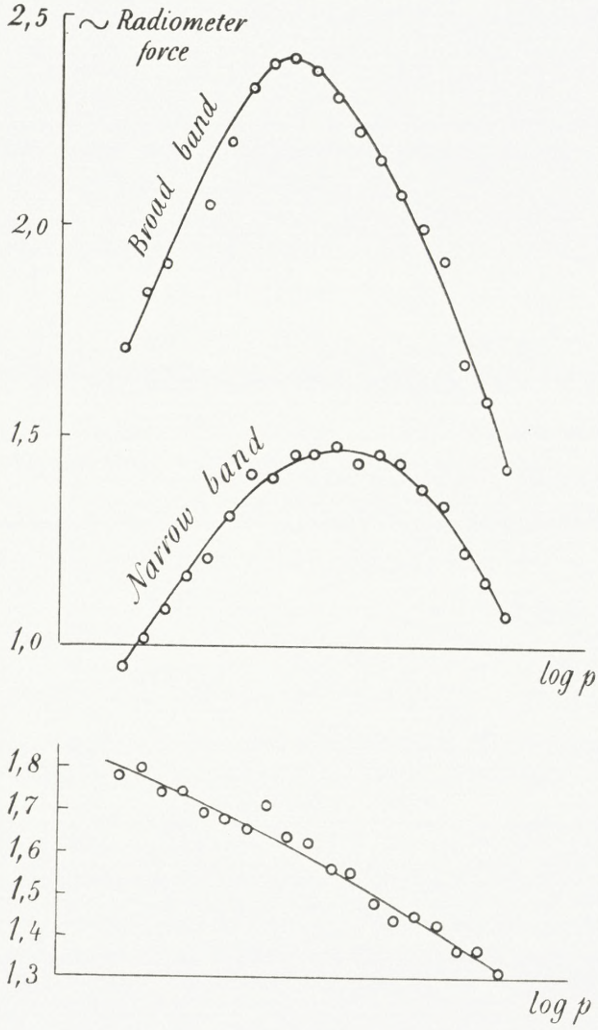

Fig. 13.

radiometer force is found to lie at a pressure, which is 1.55 times as high for the narrow as for the broad band, and the ratio between the maximum values of the radiometer forces is found to be 1.62, which again exceeds $\sqrt{2}$.

In the particulars given above, the radiometer force has been taken as proportional to the deflection of the torsion balance. This is of course only correct when the arms on which the radiometer forces exert their moments are equal. With the experimental arrangement employed, this was not quite the case. The moment of the areas of the two bands with respect to the axis of rotation was 2.127 times as large for the broad band as for the narrow band, so this must be the ratio towards which the radiometer forces converge at very low pressures. The moment of the edges of the two bands with respect to the axis of rotation was 1.028 times as large for the broad as for the narrow band, so this must be the ratio towards which the radiometer forces converge at very high pressures. The mean value of the two ratios is 1.58, which shows fairly good agreement with the value found experimentally.

The measurements with the torsion balance just mentioned have not, however, been carried out with such accuracy that they can be regarded as very reliable. These experiments must be considered to be of a preliminary kind only.

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